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ABSTRACT

A study was conducted to: (1) observe through a case study approach the exploratory behavior exhibited by 8-year-old boys and girls when calculators were made available in problem-solving situations; (2) investigate changes that occur in the kinds of arithmetical problems children construct following the introduction of calculators; and (3) observe if any changes occur in their attitudes toward problem solving after using calculators. Data were collected from protocols of children solving selected problems, from sets of original problems constructed by the children before and after a period of classroom calculator use, and from pretest/posttest responses to a questionnaire. Results show that as the children became more proficient at using the calculator throughout the school year, it became an effective aid to use while solving problems. Although an attitude change was found, it cannot be stated for certain whether the calculator influences changes related to the kinds of arithmetical problems children constructed following introduction of calculators. One recommendation offered is that the calculator be included among manipulative materials provided for student use. (Supporting documentation, including sample calculator worksheets, problems solved by children, attitudinal measure used, and protocols, is appended.) (JN)

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A STUDY OF ARITHMETICAL PROBLEM SOLVING
ABILITIES OF YOUNG CHILDREN
THROUGH THE USE OF CALCULATORS

McNicol, Mura, Lewis, O'Guy

June 1985

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"A study of arithmetical problem solving
abilities of young children through the
use of calculators"

The final report to le programme de formation
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CHAPTER 1

INTRODUCTION

1.1 General Statement

The focus of elementary school mathematics has shifted from the 'back-to-basics' movement, generally interpreted as computational ability to a rethinking of what constitutes 'basics' for today's children. In the National Council of Teachers of Mathematics' (NCTM) Agenda for Action, it is recommended that "...problem solving be the focus of school mathematics in the 1980's", and that "mathematics programs take full advantage of the power of calculators and computers at all grade levels" (p. 1). Many studies conducted in several countries, including Canada, have explored the topic of problem solving and the effect of calculator usage on the child's ability to perform computation, but few researchers have examined calculator effects on the process dimension of problem solving. Similarly, there have been relatively few studies investigating the effects of calculators on children's writing of original problems and their attitudes toward problem solving. An indication of this is evidenced in the following statement by the Gouvernement du Quebec Ministere de l'Education (1984).

At present... there is insufficient information available on the effects of calculator use on children to give any sound basis on which to

develop learning approaches or establish strategies for mathematical problem solving. Most of the research carried out so far has focused on the effect of calculator use on the child's ability to perform computations. (p. 16)

1.2 Objectives

The general aim of this project is to investigate the arithmetical problem solving abilities of eight-year-olds through the use of calculators. The specific aims of this project are as follows:

- 1) To observe through a case study approach, the exploratory behavior exhibited by girls and boys when calculators are made available in problem solving situations;
- 2) To investigate the changes that occur in the kinds of arithmetical problems children construct following the introduction of calculators; and
- 3) To observe if any change occurs in children's attitudes towards problem solving as a result of using calculators.

1.3 Definition of Terms

'Problem solving', an all encompassing term, is interpreted in this study as a basic skill. A 'problem' is a situation in which an individual is called upon to perform a task for which there is no algorithm or means apparent to that individual for solution. Thus problem solving is a means by which an individual uses previously acquired skills, concepts, and knowledge to satisfy the demands of an unfamiliar situation.

The term 'case study' will be interpreted in this report as in the definition provided by Johnson (1980). "In this type of study the researcher is interested in gathering a large amount of data from the careful observation of the behavior of a small number of subjects" (p. 21).

CHAPTER 2

REVIEW OF RESEARCH AND LITERATURE

2.1 Introduction

Although a review of research literature indicates that problem solving has received more attention than any other topic in mathematics education in the past decade, it has revealed that there is a lack of agreement as to what constitutes problem solving, what tasks are appropriate, how performance should be measured, and what key variables influence behavior. Equally revealing is a review of the literature on calculator usage in mathematics instruction. This topic has been one of the most extensively explored in the past century in general, the studies have dealt with effects on attitude, achievement, and computational skills. Suydam (1982) summarizes the results in a concluding statement in her Fifth State-of-the-Art Review when she states that "The research evidence supports the use of calculators at all grade levels, indicating that achievement will not be harmed and may be enhanced when calculators are used" (p. 10).

2.2 Problem Solving and Calculators

Numerous studies concerning problem solving and calculators have been conducted at the elementary level, and the majority report positive or neutral effects on

achievement in problem solving (Kasnic 1977; Mason 1979; Zweng 1979; Brey 1980; Shult 1980; Stewart 1980; Wheatley 1980; McNicol 1981; Szetela 1981; and Gaulin & Mura 1983).

Brey (1980) used twenty-four intact classrooms of 538 fourth grade students to investigate the effects of a planned sequence of problem solving activities and the use of calculators on problem solving and computational performance. Classrooms were assigned to two treatment groups and a control group. Both treatments were taught by their regular classroom teachers. The calculator treatment consisted of a series of sixteen problem solving lessons involving the use of calculators. The non-calculator treatment used the same series of sixteen lessons with paper and pencil for computing. Findings indicated that time spent working on story problems increased problem solving ability and computational facility. As well, the calculator did not decrease computational skill, and in fact was useful if the problems were within the child's paper and pencil ability.

In an intensive study by Stewart (1980), 146 fourth, fifth, and sixth grade students participated in an experiment to evaluate the effectiveness of a hand-held calculator when used as a computing aid during instruction in solving word problems and to evaluate the effectiveness of instructional materials which were specifically designed to be used with a calculator. A two-by-two design was used resulting in four treatment groups: (1) calculator/special materials; (2) non-calculator/special materials;

(3) calculator/standard materials; and
(4) non-calculator/standard materials. Each daily session, students received instruction consisting of ten problems to solve. A pre- and post-test were administered before and after twenty sessions of problem solving activities. A multi-variate analysis of the subscores on the pre-test resulted in a significant difference ($p < 0.01$) being found in favor of the calculator/special materials group. Results of this study suggest that using a calculator during instruction in worded problem-solving can produce significant improvement in problem solving ability in significantly less time than with paper and pencil computations. Results reported by Stewart (1980), as well as Brey (1980) tend to support the notion that calculators can be a more effective aid in problem solving for elementary children if problems are within their paper and pencil level of ability.

Using a sample of 116 grade three to eight students, Szetela (1981) found that the calculator has positive effects on the problem solving abilities of sixth graders. During the eight-week period the children were provided with calculators for use in their classrooms. For the post-test on problem solving, the students were randomly assigned to the group using calculators on the post-test or to the group using only paper and pencil. In a second section of the post-test, the groups were reversed so that all students took a test in both modes, that is using calculators in one section and using only paper and pencil in the other.

Pretest differences were non-significant. Out of eight comparisons on measures of correct answers to problems, three comparisons were statistically significant in favor of the calculator mode of problem solving. The other five differences were non-significant. On measures of problems attempted and methods correct all differences were non-significant except three, and each of these was in favor of the calculator mode.

Sztela (1981) also reported that there were no negative effects on computational ability from using the calculator. This reinforces findings made earlier by Jones (1976). He used a sample of six intact sixth grade classes (171 students) to investigate the effects of the hand-held calculator on the mathematics achievement, attitude, and self-concept of male and female students. Treatment extended over a nine-week period and consisted of routine arithmetic instruction. Experimental students used calculators; the control group used paper and pencil exclusively. Findings indicated that calculator students demonstrated significant gains at the .001 level of confidence in computation, concepts, and total mathematics achievement.

Mason (1979) added to these positive findings by reporting that no evidence showing students becoming calculator dependent could be found. Reports, such as Jones (1976) and Mason (1979), studying the effects of calculator usage on performance and attitude of elementary students are also in abundance (Bell 1978; Scandura 1978; Wilson 1978;

Meissner 1981; Standifer 1981; and Gaulin & Mura 1983). These studies have produced varied results (Suydam 1982). From her review of seventy-five calculator studies, Suydam reported that 35% of these studies provided evidence that students scored higher when calculators are used; 44% indicated that there is no significant difference; and 3% reported calculator usage resulted in lower scores than when using paper and pencil.

The assessment of problem solving ability in general has been the focus of studies by Kilpatrick (1969, 1982), Kantowski (1981), Days (1977), Greenes & Immerzeel (1982), Lee (1982), and Lester & Garofalo (1982). Focuses range from the characteristics of problem tasks and learners to the problem solving strategies utilized.

As investigations continue to be conducted with increasing frequency on the topic of problem solving and calculators, few researchers have examined calculator effects on the process dimension of problem solving. Two exceptions are Wheatley (1980) and Zweng (1979). As a result of her study of fifth grade students, Wheatley concluded that further ways to fully exploit the calculator as a tool for the learning of higher level skills must be explored. In an extensive study conducted by Zweng (1979) in grades three and four, she emphasized the need to develop a method of encouraging children to verbalize their thoughts while solving problems.

2.3 Children's Original Problems

An interesting development in the area of problem solving has been the focus on children's original word problems (Keil 1964, Graham 1978, Hungerman 1980, Wirtz & Kahn 1982 and Walter & Brown 1983). Although many different terms have been used to describe this activity, (e.g., problem constructing, problem writing, original problem writing, problem posing) the focus or intent is usually the same - it is the activity of inventing a problem and expressing it in words or otherwise. There is little doubt about the high pedagogical value of such an activity; learning to ask questions is as vital for the development of mathematical thinking as is learning to answer them. Wirtz & Kahn (1982) go as far as suggesting that training children in problem posing might improve their performance in solving problems more than training in problem solving itself. Yet very little problem posing activity takes place in the classrooms and the amount of research and publications on problem posing is miniscule compared to that on problem solving (Lembachar 1982). Hungerman notes that the advantages of this activity are twofold.

Through the writing of storyproblems, students can become more confident and motivated and begin to see mathematics as more meaningful. And teachers can learn more about their students both cognitively and affectively than they can through exclusively structured learning. (p. 9)

It is to be hoped that the recent publication of The Art of Problem Posing by Brown and Walter (1983) will encourage more work in this area.

2.4 Attitudinal Scale

It is difficult to construct self-evaluation inventories that are comprehensible and yet not transparent to children (Riedesel & Burns, 1977). This problem is compounded when working with younger children as the items on the test must be within both their readability and comprehension levels. Essay questions are simple to construct and elicit a wide range of interesting responses, but they are both difficult to quantify and often incomprehensible to younger children. Most multiple-choice and standard scaling-type items are usually transparent or they are so limited in their explanation that interpretations of items vary greatly. Many of these observations were noted by Dutton (1968), who has made a significant contribution in the area of measuring childrens' attitudes towards mathematics.

Whitaker (1980) investigated the relationships between selected noncognitive factors and the mathematical problem-solving performance of fourth-grade children. Some of the questions his study addressed are as follows :

- Question 1: Do fourth-grade students have favorable attitudes toward problem solving? ...
- Question 3: How do fourth-grade students perform on a test of problem-solving performance which provides measures of comprehension, application, and problem solving? ...
- Question 4: What is the relationship between fourth-grade students' attitudes toward problem solving and their performance in problem solving? ... (pp's. 238-239)

The findings of Whitaker's (1980) study revealed the following. For question (1), the students seemed to possess favorable attitudes toward mathematical problem solving. For question (3), the students were able to solve correctly more of the comprehension items than application items and more of the application items than problem-solving items. According to the definition of problem-solving used in this study, ("...a situation which presents an objective that an individual is motivated to achieve, although no immediate procedures are available to arrive at that objective... (p. 238)) most of the students could not be classified as good at solving problems. In question (4), the students with high problem-solving performance were found to have attitudes toward problem solving considerably higher than average, while those students with low performance have lower than average attitudes. Whitaker concluded by stating that "An interesting follow-up to the present study would be an observational investigation to determine if students actually possess the kinds of problem-solving behaviors which their responses to the problem-solving attitude scale indicate" (p. 254). It is hoped that this present study, which incorporates both the administration of an attitudinal scale and the observation of students, will fulfill this recommendation.

2.5 Case Studies

The case study approach to collecting data is by no means a new or different concept in research. Easley (1977)

has recently discussed this approach in his document, On Clinical Studies in Mathematics Education. The case study is seen by Easley as a pattern-matching task rather than focusing on a simple prediction and observational testing which is often the case in conventional experimental research. In his document, he contrasts several research strategies, discussing in detail some of the procedures and results. The perspectives and the premises of Erlwanger, Piaget, and several Soviet studies are considered in relation to the techniques and outcomes which result from their work. Easley concludes his indepth discussion of the case study approach of data collection by informing us that "Literature on methodology of case studies relevant to mathematics education is limited" (p. 28).

Easley (1977) discusses Erlwanger (1973, 1975) because of his intensive study of children's behavior in mathematics through the interview-observation approach. From his well-known case study of Benny, in Benny's Conception of Rules and Answers in IPI Mathematics, Erlwanger became aware of the immense amount of data that can be collected from the utilization of this approach.

Other researchers have applied this technique directly to the study of children's problem solving strategies and behaviors (Blake 1976, Zweng 1979, Zalewski 1974, 1980, and Hillel & Wheeler 1982). All interviewed children through the 'think-aloud' approach. Zweng (1979) reported this technique to be a total failure in her study.

First of all, the majority of children would not talk

while solving a problem. Second, those who did, talked only about how they computed, not about how they made the choice of operation.... We also tried questioning students after they solved the problem... Often the answers were meaningless.... Other answers were logical, but clearly were not the basis for the child's choice of operation... (p. 10)

Zalewski (1974) spoke very positively about the think-aloud approach when he stated "Research in general problem solving provided one essential element for this study: the thinking aloud procedure was assumed to be the most valid method of obtaining a direct record of problem solving behaviors" (p. 24). In a subsequent report on this study, Zalewski (1980) concludes that "Taped thinking aloud interviews and an associated coding system capture and classify mathematical problem-solving behaviors much better than commercial tests, but these complex interview procedures are not feasible for large scale use" (p. 140).

2.6 Video-taping/Interviewing

The video-taping of interviews has been a relatively recent trend in studies involving exploratory behavior of children during problem solving sessions. Zalewski (1974), one of the first researchers to utilize this technique, analyzes some advantages of video-taping over audio-taping. He noted that "Unspoken problem solving procedures were the most important observation noted on video tape.... Another problem solving strategy which was not readily discernable on audio-tape occurred whenever subjects drew or modified a diagram without orally indicating their exact actions" (p. 110). He concluded that "The advantages of video-tape

for recording subject behaviors in interview situations were clear without any need for statistical comparisons" (p. 110). In a later discussion of his study, Zalewski (1980) summarizes "Videotaping has a distinct advantage over audiotaping because it can detect silent rereading, drawing and altering diagrams, and written computation. Videotaped protocols also take less time to code per minute of tape" (p. 141).

Nelson & Sawada (1975) also utilized this method of data gathering in a study of problem solving behavior in young children. Here, some of the advantages of using this equipment, were noted :

There is one very obvious advantage to taped data. They represent a permanent record of the actual behaviors of the children.... The coding scheme can be worked out by observing the actual tapes and does not have to be done beforehand as would be the case if the data was coded while the behavior was going on.... Another advantage to taped data is that one is readily able to detect flaws in consistency in the use of the protocols. (p. 36)

There are, however, some shortcomings of using taped data. Nelson & Sawada (1975) note these as "The behavior of children when a camera is trained on them may not always be the same as it would be without a camera.... Another difficulty is that the behaviors have to occur in a relatively small space" (p. 36).

Not all of the observations noted above concerning video-taping are still valid today. First, children are exposed to much more technology in the form of cameras, arcade games, computers, etc. so that they are not necessarily so aware of being video-taped while solving

problems. Also the tremendous changes in technology over the past decade have brought to the market equipment with vast capabilities.

2.7 Coding

The development of checklists for the coding of protocols is indeed a slow and tedious process. There have been a number of studies in the area of problem solving in which original checklists and coding schemes were developed. (Kilpatrick 1967, Lucas 1972, Kantowski 1974, Blake 1976, Zalewski 1974).

Kilpatrick (1967) began the development of his checklist by creating a checklist of 36 questions taken from Polya's (1957) "How to Solve it". After piloting this checklist, Kilpatrick realized that not all of these processes were used and therefore reduced the list to a set of behaviors. After numerous modifications and refinements, he evolved a checklist that was used for recording a wider variety of behaviors than those sequence-coded.

Goldin & McClintock (1979), who discuss the evolution of Kilpatrick's (1967) coding scheme and relate to it the modifications made by Lucas (1972) and Kantowski (1974) as well as the somewhat different approach of Blake (1976), state that

Each of these researchers (Kantowski and Lucas) maintained (1) the checklist approach, (2) the process-sequence coding approach, and (3) major portions of the process code as well. They did, however, modify several codes and introduced others so as to capture additional processes of geometry and calculus subjects (respectively), while at the same

time making finer distinctions in such processes as deduction, use of diagrams, and use of standard processes (equations and algorithms). (p. 204)

Zalewski (1974) used a combination of the coding schemes of Kilpatrick (1967) and Lucas (1972) in his study with seventh grade students. Most of the modifications were based on the removal of the unnecessary coding symbols which were particular either to the individuals in the study or the subject area of the study. Possible alternatives to elaborate symbols were also considered.

Blake (1976) developed a scheme quite different from those of Kilpatrick, Lucas, and Kantowski. His scheme, resembling the Flanders International Analysis System, involved placing checks in the row-column blocks corresponding to the listed behaviors (row) and the order of occurrence of the behaviors (column). Goldin & McClintock (1979) summarize the differences of Blake's (1976) scheme as "...core" (mathematical content knowledge), heuristic processes, and other processes such as checking work and expressing concern are made more distinct and are located together as clusters on the coding matrix" (p. 209).

CHAPTER 3

DESIGN AND IMPLEMENTATION

3.1 Introduction

The objectives of this study are to investigate: 1) children's problem solving behavior when calculators are available, 2) variation in types and structure of problems constructed by the children, and 3) their attitudes toward problem solving after using calculators. Three types of data were collected in order to pursue these investigations: 1) protocols of children solving selected problems, 2) sets of original problems constructed by the children before and after a period of classroom calculator use, and 3) pretest and posttest responses to an attitudinal questionnaire. The study, extending over the three year period 1982-85, included the following phases: Preparation, Investigation, and Evaluation. A summary of the activities involved will be followed by a detailed description of the pilot and main studies.

Year 1 : 1982-83 Preparation

The initial phase involved the selection and training of research assistant Jeannine Lewis, and introductory meetings with Greenfield Park School Commission Educational Services Coordinator Vivian Smith, Mathematics Consultant Helen Page, St. Jude's Elementary School Principal Sean Burke, and Grade Three teacher Patricia Hill. Mrs. Hill agreed to participate in the study for two years with her

classes of 1982-83 and 1983-84.

The subsequent phase involved preparation of the materials for interview sessions, checklists and associated coding schemes, problem situations, attitudinal scales, supplementary calculator worksheets, calculators, and audio-video equipment. The remainder of the year was spent purchasing, piloting, and revising the materials. Specifications of some of the purchased materials may be found in Appendix A.

Throughout the entire year a review of the research literature was continued, and consultative visits were made with Professors Suydam, Hillel, Gaulin and St. Jude School personnel.

Year II : 1983-84 Investigation

This year was entirely school-based. Collection of data was the responsibility of the research assistants - Jeannine Lewis and Denise O'Gay - appointed for the continuation and completion of the study.

The collection of data involved: the administration of the attitudinal scale; collection of the scores of the CTBS administered by the school; the orientation of students and teachers to problem solving and calculators; observing and taping of students' behavior during problem solving sessions; the interviewing and taping of students selected for the case studies; transcribing of the taped interviews; and collecting children's original problems.

Year III : 1984-85 Evaluation

This year involved an analysis of the data compiled

(i.e., protocols, children's original problems, the attitudinal scale) and the writing of the final report.

3.2 Pilot Study

3.2.1 Preparation

During the months of November and December, 1982, research assistant Jeannine Lewis visited St. Jude School once each week to observe and interact with the grade three class during math periods devoted to problem solving. Information gathered during these sessions was used to select the children and the problems for the pilot study, and to develop the coding scheme, the interview format, and the attitudinal questionnaire.

The class participating in the pilot study was comprised of 17 boys and 4 girls. Information concerning each child's ability to verbalize, interest in the problems, and persistence in solution was taken into consideration when selecting children for individual interviews. Initially the four girls and six of the boys were selected; however, eighteen of the twenty-four children (two boys and a girl joined the class in March) were interviewed by the end of May, 1983.

Problems were developed for the individual sessions based on topics and types of problem situation deemed by the researchers to be of interest to the children, and on problems used in other studies. These included: abstract money problems, applied money problems, delivery pattern problem, travel map problem, and abstract number problems.

The objective was to try out as many types as possible in order to select the most appropriate problems for the main study.

The coding system used to analyze the children's problem solving sessions was developed after having studied previous coding systems devised by Kilpatrick (1967), Lucas (1972), Kantowski (1974), Blake (1976), Brey (1980), Zalewski (1980), and Polya (1957). The initial coding scheme developed for use during the pilot study was a 36-item behavior checklist (see Figure 1). This checklist was revised several times during the study, and after careful scrutiny in defining the purpose of the coding procedure, a different, more effective method was developed. Discussion of the coding scheme used in analysis of case study protocols is found in Chapter 4.

After a search for a suitable calculator for the pilot study, the "E.T.", a four function calculator designed for children by Texas Instruments, was selected. In January, 1983, the researchers presented the calculators to the children and subsequently gave instruction in calculator usage. Learning activities selected from various calculator workbooks and problem solving units, were presented to the class. The calculators remained in the classroom and were made available to the children at the discretion of the teacher.

3.2.2 Procedures

Individual interviews began in March, delayed by a three week teachers' strike and one week of the regularly scheduled winter vacation. The interviews with individual children took place outside the classroom in available locations in the school. Each child was asked to "think aloud" while solving each problem. This proved to be a very difficult task. Very few of the children were verbal in the individual situation, and those that did verbalize talked about what they were doing rather than what they were thinking. Each child needed to be prompted repeatedly to talk aloud, possibly interrupting the problem solving process. After several individual interviews, the decision was made to interview the children in pairs. It was anticipated that the process of the children's problem solving would be revealed by what they would say to each other. Some of the problems produced valuable verbal interaction between the children, and most of the children were interviewed in pairs during the remaining sessions of the pilot study. Two of the children, however, were found to be excellent subjects for the individual think-aloud method; subsequent sessions with these children were individual.

The sessions were audio-taped, while, at the same time, the interviewer attempted to use a checklist to record the children's observed behavior. This proved to be an inadequate means, because the interviewer had difficulty

simultaneously interacting with the children when necessary and completing the observation checklist. The audio-tapes were transcribed, but the information gathered on observed behavior was incomplete. The proposed use of video-equipment during the main study was to assure a more satisfactory means of data collection.

Once each week, the grade three class was given sets of worksheets, provided by the researchers, to aid in developing skills in problem solving with calculators.

The interview sessions continued until May, 1983. The attitudinal questionnaire was piloted, and the children were asked to create their own math problems during the last month of the preparation year.

3.2.3 Modifications

Although the children had the use of various manipulatives, in addition to the calculator, to aid in problem solving during the interviews, it was evident that the children attempted to use the calculator to please the interviewer. It was decided that for the main study the calculators would be made available from the beginning of the school year to avoid possible influence on calculator use by the association of the calculators with the researchers.

The "E.T." calculator was later evaluated to be unsuitable for the main study because of interruption in window display and inconsistency of key response to touch. Another model was selected for the main study: the Henning

Dynesen Executive 2000S, made in Denmark and distributed by GLC/Silver Burdett Publishers. It is a four function calculator in a sturdy protective case and stand, suitable for classroom use.

3.3 Main Study

3.3.1 Selection of Subjects

The main study was conducted with Mrs. Hill's 1983-84 grade three class. The class was comprised of thirty eight-year olds, 17 boys and 13 girls. The students had either limited or no experience with calculators. The new textbook series used by the class was Mathways 3 published by Copp-Clark Pittman. Throughout the study, the children were provided with supplementary calculator worksheets similar to those used in the pilot study. Selected samples of these can be found in Appendix B.

The six children, three girls and three boys, selected for the purpose of individual interviews were chosen based on a specific set of characteristics, namely the ability to:

- verbalize
- interact
- concentrate
- attempt solutions

These characteristics were considered important for exploring how calculators affect the behavior of students during problem solving sessions. At the beginning of Year 3, following a careful analysis of the collected data on the six children, a decision regarding the four case study

subjects was made. Selection of these subjects was based on the characteristics noted above, as well as the availability of a sufficient range of data.

3.3.2 Selection of Problems

Many of the problems used in the main study were adapted from those used in the pilot study and included such types as guess and check (one-step), guess and check (multi-step), role-playing, and patterns. The problems ranged in levels of concreteness and difficulty. Some of the problems included special additional manipulatives while others did not. Factors that were taken into consideration during the selection of the problems were :

- the number of operations involved in solving the problems
- the number of steps involved in solving the problems (i.e., single-step versus multi-step)
- the size of the numbers involved in the problems and in the solutions
- whether or not the children would find the problems interesting
- whether or not the calculator would be a suitable tool for use in arriving at the solution.

A copy of all problems used in this study can be found in Appendix C.

3.3.3 Development of Coding Scheme

The revisions to the checklist were not completed until the final year of the project. Because the data had been collected by means of audio-visual equipment, this provided

the researchers the benefit of extensive viewing of the data and in turn the ability to develop a checklist based upon the specifics of the study. One of the original checklists (see Figure 1) which had included all five steps to solving a problem (i.e., reading and understanding the problem, investigating and choosing a plan, setting up and solving the problem, and checking the result) was realized to contain much unnecessary information in terms of calculator usage. The number of times the child read or reread the problem did not seem to make any pertinent comment on whether or not the calculator was effective and thus many of these steps were eliminated. After repeated viewings of the available data and careful consideration of the objectives of the study, it was decided that the checklist should answer the questions:

- (1) What media were used in the solving of the problem? This could include using the calculator, paper and pencil, or counters. Both the categories of counters and paper and pencil included subcategories.
- (2) Why were they (the media) used? This involved looking at the procedures the media could have been used for (i.e., counting or algorithmic purposes) as well as the skills they could have been used for (i.e., checking, estimating, or guessing).
- (3) What was the final outcome of the problem solving session? Here, it was recorded whether or not the solution was correct or incorrect, and whether it was a partial or complete solution.

Other information that seemed important to include was the date of the session, the problem, the problem type, and whether or not the child worked individually or with a partner. Information concerning the amount and kind of

interaction between the interviewer(s) and the child(ren) was also deemed to be of importance when discussing the child's usage of the calculator.

Thus the final coding scheme varied greatly from the original version noted in Section 3.2. It was realized that in order to answer the specific research questions of this project, a specific checklist would be necessary.

The final revisions to the checklist were made as a result of using it with the protocols of the case study subjects. At this point, one of the earlier ideas used during the pilot study - using graph paper to plot the order and frequency of permutations of the behaviors - was implemented. A result of this can be seen in Figure 2. From this, the researchers were able to make some summary statements such as, "he used his calculator most for algorithmic purposes, or he did not use his counters for any purpose, etc...". It was at this point it became apparent that these graphs, along with their summary statements, could be effective in developing a chart for each child. The chart involved looking at one specific child and trying to determine whether there were any consistent changes from problem to problem over a period of time (see Figure 3). In order to try and eliminate the variable of the change resulting from a specific problem, a chart was made also for each problem. The researchers then tried to determine whether the change in a child's behavior had been consistent in that specific problem with all or most of the children (see Figure 4). The question that was considered at this

Problem Solving Observation

Steps of Problem Solving (based on Polya)

- A. Reading and understanding the problem
 - 1. reads problem
 - 2. rereads problem
 - 3. rewords problem
 - 4. reflects silently (more than 15 sec)
 - 5. asks a question (Q)
 - 6. doesn't understand
 - 7. asks for a hint (H_a)
- B. Investigating and choosing a plan
 - 1. draws a picture or diagram
 - 2. makes an organized list or chart
 - 3. recalls a similar problem
 - 4. devises a plan
 - 5. decides on operation (correct)
 - 6. decides on operation (incorrect)
 - 7. estimates solution
 - 8. asks for hint (H_b)
 - 9. gives up
 - 10. silent pause (more than 15 sec)
- C. Setting up and solving the problem
 - 1. writes a number sentence
 - 2. writes numbers in form for chosen operation
 - 3. writes numbers in meaningless way
 - 4. counts on fingers to compute
 - 5. uses manipulatives
 - 6. marks on paper to compute
 - 7. uses paper and pencil to compute
 - 8. solves problem correctly
 - 9. solution incomplete
 - 10. solution incorrect
 - 11. asks for hint (H_c)
 - 12. hint given without request
- D. Checking the result
 - 1. needs to be reminded to check
 - 2. rereads problem
 - 3. considers solution
 - 4. recalculates
 - 5. satisfied with answer
 - 6. not satisfied with answer
 - 7. asks for hint (H_d)
 - 8. method of checking (C)

Q Questions recorded C - list way of checking

H_{a-d} List hints given

Hints given

Reads problem emphasizing key words
 Repose problem in other words
 Give similar problem
 Make diagram
 Hint concerning operation action
 Personalize

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[illegible]

	Subject A	Figure 3			
	Book	Newspaper (Nov. 9/83)	Farmyard (Feb. 9/84)	Wuzzle (Mar. 29/84)	
<u>USE</u>	Why?	Does not use calculator rather <u>paper & pencil</u> for computation.	Uses it for doing <u>algorithms</u> with two numbers.	Uses it as a quick means to <u>check a guess</u> or <u>check</u> an answer that doesn't work with another media.	Uses it as a means of <u>checking a guess</u> and in finding solution.
	How?		Used with paper & pencil to compute and <u>fingers</u> for counting. Only uses with 2 numbers at a time and even breaks down 2-digit numbers into 2 parts.	used with <u>paper & pencil</u> - paper here is more for recording solutions given by calculator so he can think about what the solutions tell him.	Used alone
<u>CHANGES</u>	Use?	Does not even acknowledge the tool.	Uses only when <u>prompted</u> . Tool seems more of a hindrance than a help.	Calculator is a <u>complement</u> to <u>paper - and - pencil</u> work.	Calculator is the sole means of attaining <u>speed</u> .
	Attitude?	<u>Paper & pencil</u> is most effective media	Feels the tool is less effective (reliable) than paper & pencil computation. Seems to feel pressure to use instrument because of interviewer's presence.	Feels the calculator can be useful along with paper - & - pencil. Uses it without prompting.	Very efficient too when speed is necessary. Uses it immediately on his own without prompting.
<u>EFFECTIVENESS</u>		Does not acknowledge the calculator as a helpful or a hindering tool. Final solution not found in time allotted.	Calculator is a <u>hindrance</u> as he does not know its proper use and thus can compute quicker with paper and pencil. Final solution of problem not found by calculator.	Calculator is very <u>helpful</u> in doing the computation quickly, so that he can concentrate on what the sums & products tells him. Final solution found with help of calculator.	Calculator is a means of winning race. Is very <u>effective</u> for quick correct answers. Final solution found via calculator.

36

37

30 (c)

OCT./NOV.	Book Problem Subject B Oct. 20	Figure 4 Subject D Oct. 20	Subject A Sept. 29	
<u>USE</u> Why? How?	Uses it infrequently to check a <u>guess</u> . Used for <u>algorithmic</u> purposes. Appears to calculate mentally more effectively.	Used for <u>checking a guess</u> . Used for <u>algorithms</u> .	Does not use calculator, rather <u>paper & pencil</u> for computation.	
	Uses all her fingers and thumbs to type in numbers. Never shows a solution from her calculator.	Calculator alone.		
<u>CHANGES</u> Use? Attitude?	Uses without verbal prompting - questionable as to whether or not she feels non-verbal pressure. Uses it as a secondary tool for algorithmic purposes. (primary tool - mental arithmetic).	Lisa asks permission to use the calculator which probably prompts Michael to use it. Seems to find the calculator very helpful.	Does not even acknowledge the tool.	
	Appears to use it because of peer and interviewer's presence/pressure.	Although Michael earlier referred to using a calculator as 'cheating' he now seems to see it as important and a helpful aid.	<u>Paper & pencil</u> is most effective media.	
<u>EFFECTIVENESS</u> 38	Calculator does not appear to be very effective as her mental arithmetic of 2-digit numbers is more efficient than her inputting of addends & operations. Calculator is a <u>hindrance</u> as her mental computation is quicker. Final solution found via calculator.	Effective for doing quick algorithms. Final solution found via calculator.	Does not acknowledge the calculator as a helpful or a hindering tool. Final solution not found in time allotted.	39 39

point was whether or not the child may have changed as a result of (1) time, (2) change of partner, (3) change of problem, (4) change of problem type, or (5) a combination of any of these. Unfortunately the data collected was insufficient to make any definite conclusions. On the other hand, these charts gave the researchers some food for thought and aided greatly in the writing of the individual case studies.

3.3.4 Selection of Calculator

The calculator selected for the main study was the one decided on at the end of the pilot study. The teacher was given the calculators previous to the commencement of the school year and thus was able to integrate them in her regular classroom instruction at her discretion.

3.3.5 Attitudinal and Achievement Measures

The attitudinal measure developed during the pilot study was deemed to be sufficiently effective to use in the main study. The achievement measure, the Canadian Test of Basic Skills (CTBS), was not a planned part of this study. The school administered this test to all students during the year when the research data was being collected and thus the results were noted for the case study subjects. Caution must be exercised when interpreting the results of any standardized test and in this case the researchers wish to make it clear that there is a discrepancy between the definition of a problem according to this study and that of

the CTBS. Thus a student excelling in problem solving in the CTBS may not necessarily excel in solving problems in this study and vice versa.

3.3.6 Procedures

The main study extended over a period of nine months, from September 1983 through May 1984. During the initial visits, the children were administered an attitudinal scale (see Appendix D) and were asked to write an original word problem (see Appendix F). During the remainder of September, the children were video-taped in their classroom solving problems in small groups while having access to calculators. Following a meeting with the classroom teacher, principal, and mathematics consultant, the six children were selected for the interviews to be conducted from October through April. Although the major focus of this project was indepth case studies of these six children, all children were video-taped in a separate room on at least one occasion while they solved problems either individually or in groups of two or three. At the end of April, all the children were video-taped in their classroom while writing original word problems. Calculators were available for use at this time. In mid-May, all the children were administered the attitudinal scale and again asked to write original word problems. Throughout this entire period the classroom teacher was provided with supplementary calculator worksheets.

Data for the case studies were collected from :

- a series of interviews taped over a seven-month period and later transcribed, coded, and analyzed;
- the children's original word problems; and
- the results of a standardized test and the attitudinal scale.

The interview format was of an unstructured, informal nature. The children, interviewed either individually or in pairs, were asked to solve the problem placed before them using any of the manipulative materials available (i.e., counters, pencil and paper, calculator, ruler, or number line). Unless a need was seen for prompting or in some other way helping the student, the interviewer remained silent. There was no systematic style by which hints or help was given. An average of six interviews were conducted for each of the six students, and ranged in length from approximately fifteen to fifty minutes.

The interviews, which were held weekly and taped throughout the seven-month period, took place in a small separate room in the school during the regular mathematics class period. A Panasonic Port-a-pak colour camera and VHS recorder were used to record the children's problem solving sessions. The collection of ten two-hour tapes was kept for the purpose of coding, transcribing, and analyzing at a future time. Copies of transcribed interview sessions are included in Appendix G.

Further data collected for the study were the children's original problems. A detailed discussion of these appears in Section 4.3.

Final data collected for the study were the results of a standardized test and an attitudinal scale. The first of these, the Canadian Test of Basic Skills, was not initiated as a result of this project, rather was administered by the school. A more in-depth discussion of the attitudinal test can be found in Section 4.4 and a copy can be found in Appendix D.

CHAPTER 4

DATA AND ANALYSIS

4.1 Methodology

A discussion of many of the research methods used throughout this project can be found in the review of research and literature. Some of the well known researchers of each method are noted along with a discussion of some advantages/disadvantages of the method. A discussion of the methods inherent in the investigation of the topic of this project follows.

(a) Case Studies

The case study approach was seen to be of great value in accumulating a large amount of data on one child. Unfortunately the four case study subjects were not chosen until the final year of the project after all data collection was finished and thus at times the data for an individual child seemed insufficient. It was realized that the four case study subjects should have been chosen by the second semester of the second year so that the data collected from that period on could reflect that decision.

(b) Selection of Subjects

The criteria used for selecting subjects for individual and/or paired interviews proved to be extremely appropriate. The traits of verbalizing, interacting, concentrating, and attempting solutions were seen as key in obtaining the desired data on a child. These traits were first recognized

in the pilot study and thus it was unnecessary to spend any time establishing criteria during the main study.

(c) Selection of Problems

The problems used throughout the main study were based on the criteria discussed in Section 3.3.2. These problems were adapted from those used in the pilot study as well as those used in other studies. It was found that the calculator was used most extensively in guess and check problem types especially if the numbers and/or operations involved were beyond the child's mental and/or paper and pencil computational ability.

(d) Selection of Calculator

The calculator used in the main study was based on the decision made at the end of the pilot study. This calculator, the Henning Dyneson Executive 2000S, proved to be durable and adequate for meeting the children's computational needs. The only drawback of this calculator, in terms of being recommended for classroom use, was its cost. The increased cost, as compared with the "ET" calculator, used in the pilot study, leads one to question the advisability of its selection.

(e) Audio-Visual Equipment

The significant findings of this study resulted mainly from the usage of audio-video-equipment in collecting the data. It is thought that this is the most effective means by which to collect data because a permanent record then exists which can be viewed repeatedly. The combination of both video- and audio-taping was seen as mandatory for this project as

the repetitive viewings of the tapes facilitated accuracy in transcribing the children's verbalizations, as well as coding the children's behaviors. The wealth of knowledge that can be gained by viewing the twenty hours (approximately) of audio-video-tape is unlimited. Because of the decision to work within the stated objectives and the limited time period, the researchers of this study could not explore all the information on children, problem solving, calculators, social interaction, etc..

(f) Interviewing Technique

The interviewing technique utilized within this study was also of value in collecting the data. The 'think-aloud' approach was exercised throughout and proved to be a most valuable means by which to explore the children's behavior. In the pilot study it was realized that this technique could be facilitated through pairing children and encouraging them to discuss their problem solving processes. The interviewers spoke with the children on topics of interest before the sessions and thus believe that the children were at complete ease throughout the sessions. This helped in facilitating verbalizations and minimizing the possible distraction that could be caused by the audio-video-equipment. It seems that one of the prerequisites for success is the careful selection of subjects based on the criteria discussed in Section 3.3.1. It should also be noted that the interviewing technique of "non-interference" is one that becomes more proficient with practice. Some of the initial interviewing sessions were

found to contain too much interviewer interference and were thus not used in the analyses of the case study subjects.

(g) Coding Scheme

The coding scheme was developed to be used with the specific data collected throughout this study. It was developed in the form of a checklist that can be used with the transcripts of the problem solving sessions. It codes only the information on the transcripts which deal with the objectives of this study. For instance, information dealing with the social interaction of the children, i.e., whether it was cooperative or competitive, is not a part of the coding scheme as it was not a part of the objectives of the study. Developing the checklist within the specific framework of the objectives stated at the very beginning of the study, rather than on the wealth of data collected, was one of the most difficult aspects of this study. Because at the beginning of the project it was not realized that the children would be video-taped in pairs while problem solving, it could not be realized all the valuable data on social interaction that would be collected. Even after a checklist was developed specifically to the study's objectives, it had to be modified many times as it was used with the different transcriptions. Therefore, a final version was not completed until December, 1984.

(h) Attitudinal and Achievement Measures

The attitudinal scale devised for use within this project provided effective though limiting results. Many reasons for this were discussed in the review of research and

literature. The achievement measures administered by the school proved effective in describing the abilities of the children in the case studies.

(1) Original Word Problems

The children were asked to create their own problems without specific instruction. The problems were analyzed and categorized as discussed in Section 4.3.

4.2 Case Studies : Introduction

The main objective of this study was to observe, through a case study approach, the exploratory behavior exhibited by girls and boys when calculators are made available in problem solving situations. Throughout the rest of this section, the case study subjects will first be given a general description, that is, their age, mathematical ability and the number of times interviewed. Next, a discussion of the following questions will occur:

- Why and how did the child use the calculator?
- Was there any apparent change in the child's use of the calculator and the child's attitude toward the calculator?
- Was the calculator considered by the child as an effective medium to use in mathematical problem solving?

The following problems were selected for use with the case study subjects. The month(s) in which each problem was presented to the child(ren) is noted afterwards in brackets.

Book Problem (October)

Open the book so that the sum of the pages is 89. 155. (A book was provided for the child(ren) to manipulate)

Newspaper Problem (November)

Julie delivers newspapers from Monday to Saturday. On Monday she delivers 27 newspapers. If each day she delivers 10 more newspapers than the day before, how many newspapers

does she deliver on Saturday? (How many does she deliver in a whole week?)

Farmyard Problem (January- February)

In the farmyard there are 18 animals. Some are chicken and the rest are sheep. If there are 58 (46) legs altogether, how many of the animals are chickens and how many are sheep?

Signs Problem (January)

Put +, -, x, or - in the boxes to make the number sentence true.

$$64 \quad 8 \quad 5 = 3$$

$$37 \quad 8 \quad 166 = 130$$

Wuzzle Problem (March)

You are going to make some Wuzzles to sell at the Spring Fair. Yarn for one Wuzzle costs 10 cents. Eyes cost 5 cents each. Pipecleaners cost 3 cents each. How many Wuzzles can you make for \$5.00? (How much would you sell them for? How much extra money would you make?) (The children were provided with an already prepared "Wuzzle")

4.2.1 Case Study A

A is perceived to be an extremely bright boy and throughout the study he was curious, keen, intuitive, and self-confident. His skills in mathematics seemed very exceptional for his age which at the time when the study began (1983-10-20), was 9.0 years.

In the results of the CTBS' Mathematics Section administered in November 1983, Subject A ranked in the 99th percentile. This represents a grade equivalence of the ninth month of the fourth grade. This total mark was comprised of a 97th percentile ranking in concepts; 91st percentile ranking in problems; and a 99th percentile ranking in computation. In the total composite score over all subject areas, A ranked in the 78th percentile, which represents a grade level equivalence of the ninth month of the third grade. In the class marking system for the entire year this student received an A for both his mathematics ability and his effort.

Subject A was video-taped in a separate room on six occasions - three times individually and three times with a female partner. He worked very well alone, but when paired with another student often attempted (and sometimes succeeded) in dominating the situation. Subject A's resilient nature and his appreciation for mathematics made him an excellent subject for the exploration of problem solving behaviour.

Use of Calculator

A's usage of the calculator changed over the period of the four sessions in which he used the calculator. Initially, in the Book and Newspaper problems, he did not use his calculator at all. By November he was using his calculator for computational purposes only. This usage expanded by February to include computation in the form of guess and check, and checking a solution already found via

another medium. Both of these uses were exercised throughout the rest of the study.

The means by which A used his calculator also changed over the period of the four sessions. In the Book problem A did not acknowledge his calculator. He acknowledged it in the Newspaper problem (with some prompting), but his usage was extremely limited as he found paper and pencil computation and finger counting to be more effective. He showed a lack of understanding of the calculator's capabilities when he split any two-digit number into parts before adding and added two digits only in his calculator at a time. By the time he did the Farmyard problem A had begun to use his paper and pencil in close conjunction with his calculator. However, during the Wuzzle problem the calculator was used exclusively.

Changes

There was quite an apparent change in A's use of the calculator as may be noted from the above discussion. The uses ranged from non-use (Book problem), to partial-use after a prompt was given (Newspaper), to use as a complement to paper and pencil (Farmyard), to use as an instrument that provides speed when necessary (Wuzzle). In each of the last two problems the calculator was used without any form of prompting whatsoever.

The change in calculator usage over the four sessions was strongly reflected in the attitude observed throughout these sessions. During the Book problem the calculator was totally ignored and in its place paper and pencil

computation was used. It seems as if this method of computation was more reliable. During the Newspaper problem this attitude was confirmed and even justified when it became apparent how limited his knowledge was of the calculator's capabilities. Due to the ineffective style used to compute a list of numbers on the calculator, it was surely more effective to compute by paper and pencil. After some instruction was given concerning the calculator's capabilities, A's attitude changed to include many uses. He used the calculator both as an instrument to do the 'nitty-gritty' computation while he thought about the solution process, and also as an instrument to offer him speed in calculating so that he could find the solution to the problem before his partner did.

Effectiveness

Following some instruction on the capabilities of the calculator it seemed as if A's attitude changed from feeling the calculator was a hindrance to feeling it was a great help. By the end of the study the calculator was effective in attaining the final solution in the manner or the time frame in which A worked.

4.2.2 Case Study B

B is perceived to be a very bright girl. She is exceptionally competent at mental computation in mathematics but due to her strong desire to always finish first she sometimes loses accuracy. At the time when this case study began (1983-10-20), she was 8.6 years of age.

Subject B ranked in the 99th percentile in the Canadian Test of Basic Skills' (CTBS) Mathematics Section written in November 1983. This represents a grade equivalence of the beginning of the fifth grade. The total mathematics mark was comprised of a 93rd percentile ranking in concepts; a 99th percentile ranking in problems; and a 93rd percentile ranking in computation. In the total composite score over all the subject areas, B ranked in the 99th percentile which represents a grade equivalence of the beginning of the fifth grade. In the class marking system for the entire year this student received an A for both her mathematics ability and her effort.

Subject B was video-taped in a separate room on six occasions - once individually, four times with a male partner, and once with a female partner. She is both intuitive and retentive when solving problems. She is also critical, especially of others' strategies, and thus she works best at problem solving when paired with another person. B did not enjoy working alone when solving a problem as she worked best when she could evaluate and criticize her partner's strategies which in turn helped her see how the problem could be solved. Without this type of interaction it did not seem as if B pushed herself hard enough to arrive at a solution to a perplexing problem.

Use of Calculator

B did not show any great change in her use of the calculator until the third problem (Wuzzle). In the first and second problems (the Book and the Newspaper), the

calculator was used infrequently to check a guess or to computation. In the Newspaper problem there is some question as to whether the calculator was actually being used to help find the solution to the problem or whether it was being used to feign working towards the solution.

B never effectively coordinated the use of her calculator with any other media. Her actual fingering while using the calculator did not improve either over the three problem solving sessions in which the calculator was used. However, it did seem that her level of confidence or ability to get the correct answer by using the calculator improved by the time she worked on the Wuzzle problem (end of March). Here, where she worked with a very competitive partner, she used the calculator exclusively and was able to obtain the correct solution.

Changes

There was a definite change in use from the Book and Newspaper problems to the Wuzzle problem. It is not clear whether this was an actual change in usage of the calculator or whether the specific Wuzzle problem prompted a need for quick effective action. At no time was a verbal prompt given, but in both the Book and Newspaper problems it is questionable as to whether B felt pressure to use the calculator either because her partner used it or because the presence of the interviewers suggested her using it. These same pressures may have been existent in the Wuzzle problem but it seems as if the challenge to be first in finding the correct solution far exceeded anything else.

There was no apparent change in attitude over the period of the three sessions in which the calculator was used. In both the Book and Newspaper problems there seemed to be little pressure to race to get the answer and thus the calculator was 'fiddled with' in hopes that it would produce the answer. In the Wuzzle problem the calculator became a very valuable aid because it was the tool for getting a quick, correct answer. The question of whether she would use the calculator in such an efficient manner if she were put in a situation like the Book or Newspaper problem again is difficult to answer. Whether or not Subject B's attitude actually changed cannot be clearly determined. It is possible that, (a) her attitude actually changed so that she later accepted the calculator as being helpful, or (b) she became more proficient at using the calculator during the study and therefore came to see its value, or (c) she could use the calculator proficiently all along and thus would likely continue with the same inconsistent use as was displayed throughout this study.

Effectiveness

The calculator did not seem to be helpful to B in the Book problem as her ability to compute answers mentally was more efficient. In the Newspaper problem the calculator was totally ineffective in helping obtain the solution to the first part of the problem, but it was somewhat effective in correctly adding up a list of numbers in the second part of the problem. In the Wuzzle problem the calculator was very effective in helping to obtain the correct solution as B's

attitude toward the calculator and use of it was much more mature. No longer did she aimlessly tap away at the keys as if the solution would magically appear. In the Wuzzle problem she only touched the calculator keys when she knew exactly what she wanted to input and how to do it.

4.2.3 Case Study C

C is perceived as being of average intelligence for her age. Her skills in mathematics were somewhat limited by her concrete level of development but she compensated nicely for this through extensive use of the available manipulatives. At the time when this case study began (1983-10-20), she was 8.8 years of age.

Subject C ranked in the 34th percentile in the Canadian Test of Basic Skills' (CTBS) Mathematics Section written in November 1983. This represented a grade equivalence of the ninth month of the second grade. The total mathematics mark was comprised of a 40th percentile ranking in concepts; a 41st percentile ranking in problems; and an 18th percentile ranking in computation. In the total composite score over all subject areas, C ranked in the 66th percentile which is equivalent to the sixth month of the third grade. In the class marking system for the entire year this student received a B for mathematics ability and an A for effort.

Subject C was video-taped in a separate room on seven occasions, always with male partners. Although her level of concreteness could be seen as a limitation in problem solving, she quite ingeniously compensated through

creatively using her counters. C worked especially well with her partners as she was interactive, tolerant, organized, and resourceful.

Use of Calculator

There was an observable change in C's use of the calculator throughout the four problem sessions. In early sessions (Newspaper and Farmyard problems) the calculator was not acknowledged at all whereas later the calculator was used either for guess and check, checking a solution found by another medium, or computational purposes.

During the two problem sessions in which C used the calculator (Signs and Wuzzle problems) it was never used exclusively. She originally started to solve the Signs problem with pencil and paper and only used the calculator to check her solution. For the second part of the problem she used her calculator for guess and check while manipulating the signs and symbols given to help solve the problem. By the end of the study she had increased the number of media used to help solve the problem to include her counters, fingers, and paper and pencil. All of these media were used in close conjunction with her calculator.

Changes

The most noticeable change occurred midway through the study when C started to use her calculator to help her solve problems. The calculator seemed to be too abstract a medium to 'think with'. On the other hand, her counters, which were used extensively in the Farmyard and Wuzzle problems, helped her think at a much more concrete level. Of the two

problems in which C used her calculator (Signs and Wuzzle), she was prompted only in one.

There was a small change in C's attitude toward the calculator over the period of the four problem sessions. During the Newspaper and Farmyard sessions she saw no apparent use for the calculator in terms of helping her obtain a solution, whereas in the last two problems, Signs and Wuzzle, she saw how the calculator could do some computation that she would otherwise have to do mentally or by paper and pencil. She did not change her attitude to the point where she felt the calculator could be used exclusively in solving a problem. It became apparent that this might be related to her level of concreteness.

Effectiveness

In all the problems in which the calculator was used it was effective as a computational tool. As noted earlier, this effectiveness was limited to simultaneous usage with manipulative material(s) only.

4.2.4 Case Study D

D is perceived to be of average intelligence for his age. He seemed to be very competent in mathematics skills such as problem solving and computation. At the time when this case study began (1983-10-20), he was 8.11 years of age.

Subject D ranked in the 97th percentile in the CTBS' Mathematics Section written November 1983. This represented a grade equivalence of the sixth month of the

fourth grade. This total mathematics mark was comprised of an 87th percentile ranking in concepts; a 96th percentile ranking in problems; and a 99th percentile ranking in computation. In the total composite score over all the subject areas, D ranked in the 91st percentile which is equivalent to the fourth month of the fourth grade. In the class marking system for the entire year this student received an A for both his mathematics ability and his effort.

Subject D was video-taped in a separate room on six occasions, always with female partners. D's keen and curious nature made him an interesting subject for a problem solving study. His tolerance, resilience, flexibility, and interactiveness also allowed him to work well with other students.

Use of Calculator

D's use of the calculator was very consistent during the five problem solving sessions (Book, Newspaper, Farmyard, signs, and Wuzzle). His usage included checking a solution found by another medium, guess and check, and computation. Over the five sessions there was no pattern showing a development in his usage of the calculator. He used the calculator for computation in all five problems, and used the calculator to check a previously found solution in only two of the problems.

The most popular medium used with the calculator was paper and pencil. In the only session when paper and pencil was not used with the calculator (Book problem), the

calculator was used alone. D's level of confidence in using the calculator wavered inconsistently throughout the sessions.

Changes

D's use of the calculator changed little over the five sessions. During the Book and Newspaper problems he was probably influenced by his partner to use the calculator. In the first instance, she asked permission to use her calculator which could in turn have caused D to use his. In the second instance D was trying to justify his paper and pencil answer to his partner when she stated, "That makes no sense...put that in your calculator - it'll go berserk!". While trying to solve the Farmyard problem it was D who suggested he use the calculator while she used her counters. Again in the Signs problem D was prompted by the interviewer to check his paper and pencil solution and in turn he began using his calculator. During the last problem, the Wuzzle, D used his calculator without prompting but had much difficulty as he had not had previous experience using the decimal point for monetary calculations. After instruction was given, he continued using the calculator and finally solved the problem.

There is some noticeable change in attitude over the period of the five problem sessions, but the greatest change is evidenced by a comment made by D during one of the early class visits in September, when upon being offered a calculator to help him solve a problem, D replied, "We don't want to cheat!". Once he realized that calculators were not

considered 'cheating devices', D was the most consistent of all subjects in his usage of it. Over the five sessions there was a noted change in the understanding of the calculator's capabilities and this in turn changed D's attitude towards it. In the Newspaper problem, D showed a lack of understanding that the calculator is only 'correct' if the correct numbers are inputted. When he inputted his incorrect paper and pencil solution where he had added one extra ten and got the same solution from the calculator as when he had added the numbers on paper, he said to his partner, "...even says in the calculator". This attitude seemed to persist in the Farmyard problem where D continued using his calculator when he had absolutely no idea what to input or what to expect. It seems his thinking here was somewhat similar to Subject B's in that he seemed to believe that just punching in the numbers given in the problem would in some way magically make the correct solution appear.

Effectiveness

There was quite a range in the effectiveness of the calculator over the five problems. In problems like the Book, the second part of the Newspaper, and the Signs where an equation to input into his calculator was clearly seen, the solution was found very effectively by D. On the other hand, in a problem like the Farmyard where many factors had to be taken into account and therefore no one equation represented the solution, D was at a total loss as to how the calculator could help and thus helped his partner solve it with counters. In the Wuzzle problem, which is almost like the Book, Newspaper, or Signs, except that it is a multi-stepped guess and check, D kept trying multiplication equations to see which product came closest to the number stated in the problem. This once again was an effective manner in which D could use his calculator.

4.3 Children's Original Problems

4.3.1 Introduction

The objectives of this aspect of the study were (a) to investigate the changes that might occur in the kinds of problems constructed by the children after having been acquainted with calculators and having had access to calculators for several months, and (b) to investigate the differences that might exist between the problems constructed by girls and those constructed by boys. The design of the present study involved asking the children to produce word problems twice; once towards the beginning of the school year and once towards the end.

The first problem writing session took place on September 13, 1983 and the second one on April 12, 1984. Because the instructions of the second session were unclear, it was decided that a third session should be held. It took place on May 10, 1984. The data from all three sessions follow.

It is usually suggested that when students are asked to construct a problem they should be supplied with some kind of starting point: this might be a real life situation or a geometric situation about which the students must ask a question, an arithmetical identity about which the students must write a story and so on. In this particular study, no starting point was provided. The children were simply asked

to "write a word problem". They were also asked to solve their own problems (which many of them did). Many of the children produced more than one problem, and most of them illustrated their problems with drawings.

4.3.2 Data and Discussion

Twenty-eight children were present at the first problem writing session; altogether they constructed 36 problems. At the second session, 22 children were present and they constructed 64 problems altogether (a twenty-third child was present, but did not sign his or her name and was dropped from the analysis). Only one of the children present at the second session was not present at the first session (a boy). At the third session 22 children were present and they produced 25 problems altogether (again a twenty-third child who was present but did not sign his or her name was dropped from the analysis). Only one of the children present at the third session was not present at the first one (a boy); while five of the children present at the third session were not present at the second one (three girls and two boys). Thus 17 children were present at all three sessions, 21 were present at sessions 1 and 2, and 21 were present at sessions 1 and 3. Table 1 gives the distribution of the children by sex and by number of problems constructed at each session.

Table 1

	<u>First session</u>		
	Boys	Girls	Total
number of children who produced one problem only	13	9	22
number of children who produced two problems	2	2	4
number of children who produced three problems	1	1	2
total number of children	16	12	28
total number of problems	20	16	36

Second session

	Boys	Girls	Total
number of children who produced one problem only	3	1	4
number of children who produced two problems	4	5	9
number of children who produced three problems	3	1	4
number of children who produced four problems	3	0	3
number of children who produced five problems	1	0	1
number of children who produced thirteen problems	0	1	1
total number of children	14	8	22
total number of problems	37	27	64

Third session

	Boys	Girls	Total
number of children who produced one problem only	12	7	19
number of children who produced two problems	1	2	3
total number of children	13	9	22
total number of problems	14	11	25

All three sessions together

	Boys	Girls	Total
Total number of problems	71	54	125

ANALYSIS

The problems were analysed by the sex of the children who wrote them, according to the following variables: length, syntactic structure, verbal context, sex of characters, numbers' size, and mathematical content and type.

LENGTH

Length was measured as the number of written words in the problem statement.

Table 2
First session

	Boys	Girls	Total
Mean length	10	10	10
Minimum length	2	7	2
Maximum length	18	16	18

Second session

	Boys	Girls	Total
Mean length	16	17	16
Minimum length	8	10	8
Maximum length	27	24	27

Third session

	Boys	Girls	Total
Mean length	20	18	19
Minimum length	7	10	7
Maximum length	47	26	47

Four problems from the second set have been omitted from the length analysis because of their non-verbal format (histograms or drawing); two of these problems were produced

by girls and two by boys.

There is little difference between girls' and boys' problems in length: in each session both the shortest and the longest problems belonged to boys, the slightly higher mean length of boys' problems in the third session is due to two boys producing exceptionally long problems. On the other hand there is a 90% increase in the mean problem length between the first set of problems and the third one.

SYNTACTIC STRUCTURE

The problems have been classified by syntactical structure as follows:

(a) Problems consisting of main clauses only.

(a).1. Problems with two main clauses, both clauses complete.

(a).2. Problems with two main clauses, at least one incomplete clause.

(a).3. Problems with three main clauses, all clauses complete.

(a).4. Problems with three main clauses, at least one incomplete clause.

(a).5. Problems with more than three clauses, all clauses complete.

(a).6. Problems with more than three clauses, at least one incomplete clause.

(b) Problems including at least one subordinate clause.

(b).1. Problems including a hypothetical clause (use of "if").

(b).2. Problems including at least a subordinate, but no hypothetical ones.

(c) Problems presented in a nonverbal format (drawing or hystogram).

An incomplete clause is one whose verb is missing (telegraphic style). Example:

"5 apples and 2 were eaten how many were left?"
 "5 apples" was interpreted as an incomplete main clause,
 meant to be "There were 5 apples."

Problems including a hypothetical clause have been singled out because this expression form seemed to be a particularly sophisticated one for children at the third grade level.

Table 3
First Session

Syntactic structure	Boys	Girls	Total
Two main clauses incomplete	3	0	3
Three main clauses complete	4	8	12
Three main clauses incomplete	10	6	16
Total three clauses	14	14	28
More than three incomplete	1	0	1
Total main clauses only	18	14	32
Hypothetical clauses	0	2	2
Other subordinate clauses	2	0	2
Total number of problems	20	16	36

Second Session

Syntactic structure	Boys	Girls	Total
Three main clauses complete	21	16	37
Three main clauses incomplete	2	0	2
Total three clauses	23	16	38
More than three clauses complete	9	8	17
Total main clauses only	32	24	56
Hypothetical clauses	2	1	3
Other subordinate clauses	1	0	1
Non verbal formats	2	2	4
Total number of problems	37	27	64

Third Session

Syntactic structure	Boys	Girls	Total
Three main clauses complete	5	3	8
Three main clauses incomplete	3	1	4
Total three clauses	8	4	12
More than three complete	3	5	8
Total main clauses only	11	9	20
Hypothetical clauses	1	1	2
Other subordinate clauses	2	1	3
Total number of problems	14	11	25

Out of 125 problems, 108 (86%) consist of main clauses only, and 78 (62%) consist of exactly three main clauses. The percentage of problems including more than three clauses and/or a subordinate clause increases from one session to the next (14%, 33%, and 52% respectively). Possible explanations for problems comprising three or more clauses may be the requirements of its mathematical structure (multiple-step problems), or simply the desire for a more elaborate context. An example of the latter possibility is:

"There was a sail outside the house I bought 4 pencils for 10 cents and I payed 25 cents how much will I get back"

In this case the first clause does not play any functional role: it could be dropped without altering the mathematical structure of the problem.

The percentage of problems with incomplete clauses decreases dramatically from the first session to the second one, but increases again in the third session (56%, 3%, 16% respectively). This might be due to the fact that the instructions were unclear in the second session.

Boys were more likely than girls to produce problems with incomplete clauses: out of 26 such problems, 19 were

written by boys and 7 by girls. In other words, 27% of the problems constructed by the boys contained at least one incomplete clause, while only 13% of those constructed by the girls did. Apart from this aspect, there are no apparent differences in syntactic structure between the problems written by boys and girls.

VERBAL CONTEXT

Problems have been classified according to their verbal context in the following categories:

(a) Inanimate objects (knives, nails, balls, clocks, books, pencils, rulers, dolls, toys, video games, flowers, stars). These objects may be coming and going (!), given, received, bought, lost and found, or there may be no specified action, e.g., "102 nails in a box, 209 nails in the other box. How many all together?"

(b) Money: found, lost, given, received. This category comprises problems concerning money by itself: problems concerning money paid to buy something are classified according to what is bought (objects, food, or animals).

(c) Vehicles (cars, boats, trucks, spaceships): simply moving or without any action being specified.

(d) Vehicles in a violent situation: boats and planes in war, planes shot down, boats sunk, spaceships blown up.

(e) Food (eggs, candies, apples, cookies, carrots, lollipop, gum, pizza, hotdogs, cans of soup). The food is "going", given, eaten, bought, or simply owned.

(f) Animals (cats, dogs, birds, butterflies, horses,

snakes). The animals are moving, given or bought, or not involved in any specified action.

(g) Team games (football, basketball, hockey, baseball, soccer). Players coming and leaving, getting penalties, getting kicked out of the game, scoring points. Watching a game. Puck travelling so many meters.

(h) Playing (playing tag, playing ball, making snowmen)

(i) People or children coming and going, getting on/off buses and streetcars, having dinner, going to the movies, going to a plane show, going to the zoo, or not involved in any specified action (e.g., age problem).

(j) Fisherman catching fish, mousecatcher catching mice.

(k) People in conflict situations (punching, knocking out, failing to kiss).

TABLE 4

Problems from all three sessions together

Context category	Boys	Girls	Total
(a) Objects	12	19	31
(b) Money	0	6	6
(c) Vehicles	8	8	16
(d) Vehicles and violence	5	0	5
(e) Food	11	7	18
(f) Animals	4	3	7
(g) Team games	13	1	14
(h) Playing	1	5	6
(i) People	12	5	17
(j) Fishermen, mousecatcher	3	0	3
(k) People in conflict	2	0	2
Total	71	54	125

There appear to be some differences between the contexts produced by boys and girls. All the problems in categories (d), (j), (k), i.e., all problems presenting some element of violence or conflict, including the relatively minor violence of catching fish or mice, belong to boys. Also, with one exception, all problems involving team games (category (g)) belong to boys. On the other hand, girls are the only ones to have produced problems about money without further context (however 19 of the problems in categories (a), (e), and (f) also involve money: 8 of them written by boys and 11 by girls). Further sex-related differences may be hidden by the classification chosen, for instance, girls are the only ones to have written about dolls and generic "toys", while boys are the only ones to have written about knives, nails, balls, and video games.

SEX OF CHARACTERS

Problems involving human characters have also been classified according to the sex of the characters mentioned, as follows:

(a) male characters only: dad, brothers, fisherman (further referred to as "he"), male names, I (produced by a boy), Santa, a guy, etc.;

(b) female characters only: mother, aunt, female names, I (produced by a girl), etc.;

(c) both male and female characters;

(d) some male characters and some characters whose sex is not specified;

(e) some female characters and some characters whose sex is not specified;

(f) characters whose sex is not specified: kids, children, people, friends, etc.;

(g) teams, players

TABLE 5

First session

Sex of characters	Boys	Girls	Total
(a) male only	8	2	10
(b) female only	0	1	1
(c) male and female	0	1	1
(d) male and unspecified	0	0	0
(e) female and unspecified	0	0	0
(f) unspecified	3	1	4
(g) team	1	0	1
Total number of problems involving human characters	12 (60%)	5 (31%)	17 (47%)

Second session

Sex of characters	Boys	Girls	Total
(a) male only	11	2	12
(b) female only	0	17	17
(c) male and female	3	3	6
(d) male and unspecified	5	0	5
(e) female and unspecified	0	1	1
(f) unspecified	7	2	9
(g) team	2	0	2
Total number of problems involving human characters	28 (76%)	25 (31%)	53 (83%)

Third session

Sex of characters	Boys	Girls	Total
(a) male only	5	0	5
(b) female only	0	5	5
(c) male and female	1	1	2
(d) male and unspecified	1	0	1
(e) female and unspecified	0	2	2
(f) unspecified	4	2	6
(g) team	1	0	1
Total number of problems involving human characters	12 (86%)	10 (91%)	22 (88%)

Problems from all three sessions together

Sex of characters	Boys	Girls	Total
(a) male only	24	4	28
(b) female only	0	23	23
(c) male and female	4	5	9
(d) male and unspecified	6	0	6
(e) female and unspecified	0	3	3
(f) unspecified	14	5	19
(g) team	4	0	4
Total number of problems involving human characters	52 (73%)	40 (74%)	92 (74%)

In the first session, proportionally fewer problems involving human characters were produced than in the second and third sessions (47% vs. 83% and 88% respectively). Overall girls and boys produced about the same percentages of problems involving human characters.

For both boys and girls the largest category is by far that of problems with characters all of the same sex as the child. While no boy produced a female-only problem, girls produced four male-only problems. Boys also produced more problems with characters of unspecified sex.

Few children created problems involving both female and male characters. Two of these seem worth quoting:

"There were 4 girls Michael J. kissed 3 how many didn't get kissed"

(The story is accompanied by a drawing of a boy, three smiling girls, and a fourth girl in tears apart from the rest.)

"The boys scored 8 points. The girls scored 6. How many more points did the boys score?"

(The story is accompanied by the drawing of a board displaying the boys' and girls' scores and announcing the boys to be the "winner".)

NUMBERS' SIZE

This is one of the features of the children's problems that was found to be most interesting. The following is an analysis of the numerical data appearing in the problem statements, whether presented as numerals or as number words (numbers occurring in the solution of the problems are not included in the analysis). There are only six instances of decimal numbers (dollars and cents); all six are from the second set of problems, five of them were produced by girls and one by a boy. All the rest of the numerical data produced by the children were whole numbers. For clarity, the latter set has been broken into four intervals: numbers between 1 and 5, numbers between 6 and 10, numbers between 11 and 99, and numbers over 100.

Table 6 gives the distribution of the numbers produced by the children by intervals and by sex. It also gives the median and the maximum number produced (in this case the median is more interesting than the mean because there are a few instances of numbers much larger than all the rest).

TABLE 6

First session

Interval	Boys	Girls	Total
1 - 5	18 (43%)	24 (75%)	42 (57%)
6 - 10	12 (29%)	6 (19%)	18 (24%)
11 - 99	10 (24%)	2 (6%)	12 (17%)
over 100	2 (5%)	0 (0%)	2 (3%)
Total	42	32	74
Median	6	5	5
Maximum	209	20	209

Second session

Interval	Boys	Girls	Total
1 - 5	23 (29%)	14 (27%)	37 (29%)
6 - 10	16 (21%)	14 (27%)	30 (23%)
11 - 99	36 (46%)	20 (39%)	56 (17%)
100 - 999	0 (0%)	3 (6%)	3 (2%)
over 1000	3 (4%)	0 (0%)	3 (2%)
Total	78	51	129
Median	10	10	10
Maximum	3000	295	3000

Third session

Interval	Boys	Girls	Total
1 - 5	4 (14%)	7 (25%)	11 (19%)
6 - 10	6 (21%)	3 (11%)	9 (16%)
11 - 99	13 (45%)	12 (43%)	25 (44%)
100 - 999	1 (3%)	2 (7%)	3 (5%)
over 1000	5 (17%)	4 (14%)	9 (16%)
Total	29	28	57
Median	20	17	20
Maximum	100051200	20000	100051200

The size of the numbers produced by the children does increase considerably from the first session to the second one and from the second session to the third one. This can be seen by observing that the percentage of the numbers in the 1-5 interval decreases from one session to the next, while the percentage of numbers in the 11-99 interval increases as does the percentage of numbers over 100. The median number produced also increases from session to

session along with the maximum number produced, reaching an amazing nine-digit number in a problem written during the third session - this number could not even be displayed on the eight digit screen of the children's calculators! (It must be noted though that the boy who produced it did not even attempt to solve his problem.)

At the time when they wrote the first set of problems, the children, who did not all come from the same second grade class, had been taught two-digit numbers as large as 99. Most children, however, chose to use rather small numbers; 81% of the numbers produced do not go beyond 10. One child used two three-digit numbers in his problem and added them correctly.

By the time the children wrote the second set of problems, they had been taught numbers up to about 300. However 52% of the numbers produced were still not greater than 10. In the third set of problems the percentage of numbers in this category goes down to 34%. The second set of problems includes three instances of four-digit numbers, occurring in two problems produced by boys; both problems were solved correctly. The third set of problems includes six problems featuring numbers with four, five, six or nine digits; however, none of these problems was solved correctly. Four of these six problems with large numbers were produced by boys and two by girls.

In the first set of problems, there is a tendency for girls to produce smaller numbers than boys; however the same tendency is not evident in the second and third set. In all

three sets the largest number used was produced by a boy.

MATHEMATICAL CONTENT AND TYPE

Except for four problems about hystograms, all problems are arithmetical ones. They have been classified into the following types:

1. Single-step problems

1.1 $A+B$ with a static condition described.

Example: "In the parking lot there are 90 cars on one side and the other side 50. How many cars in all?"

1.2 $A+B$ with a described transformation. Example:

"22 kids were playing tag 4 more came along how many altogether"

1.3 $A-B$ with a described transformation. Example:

"10 candy in the box I ate 3 how many candy are left?"

1.4 $A-B$ with a static condition described.

Example: "Tom is 10 years old his brother is 2 years old how many years older is Tom then his brother"

1.5 $A \times B$ Example: "If one can of soup cost 65

cents and you have to get 7 cans how much money do you need?"

2. Multiple step problems

Example: "35 people wanted to play basketball. 13 more people came. 3 went away. then they went into groups of five. How many groups of five are there?"

The above classification is a modified version of the one proposed by Gerard Vergnaud (1981). Type 1.1 (addition with a static condition) corresponds to Vergnaud's first category of additive relations (two measures combined to produce a measure). Types 1.2 and 1.3 (addition and subtraction with a transformation) both correspond to Vergnaud's second category of additive relations (a transformation applied to a measure produces a measure): it was thought to be useful to split this category in two in order to distinguish addition and subtraction. Type 1.4 (subtraction with a static condition) corresponds to Vergnaud's third category of additive relations (a relation is established between two measures). None of the children's problems fall into Vergnaud's fourth, fifth, and sixth categories of additive relations (two transformations combined to produce a transformation, a transformation applied to a relative state, and two relative states combined to produce a relative state). All of the children's problems solvable by multiplication (type 1.5) belong to Vergnaud's category of measure isomorphisms.

Table 7 presents the distribution of problems by mathematical type and sex. A few problems in each set could not be classified; these are problems without a question, problems with an unrelated question or otherwise poorly constructed problems. Examples:

"I went to a baseball game and the Expos won 30 to 19 and the Expos won",
 "There are 11 planes in the sky 3 got shot down Who won the war?"
 "30 apartments in each apartment 1234 people in all",

"10 boys are play tag 5 boys left How mane more "
 "people got of 5 people got on How much peapl got
 on."

This does not imply that all the problems that could be
 classified by mathematical type were perfectly well
 constructed. Two frequent faults are the omission of
 temporal or other referential marks and of the verb "to be".

Examples:

"20 kids on the bus. 10 kids get on how many on the
 bus?"

"One team there are 90 men. 60 plars got kicked out
 of the game. How many men are there?"

"13 cars are in the parking lot 9 left how many
 left?"

In the last example the omission of the verb "to be"
 completely changes the meaning of the question sentence: the
 solution worked out by the child indicates that he meant to
 ask "How many cars are left?".

Table 7

First session

Mathematical type	Boys	Girls	Total
A+B static condition	1	0	1
A+B transformation	3	2	5
A-B transformation	9	11	20
A-B static condition	1	0	1
total of single-step problems	14	13	27
A+B+C	1	0	1
A-B-C	1	0	1
total of multi-step problems	2	0	2
problems with incomplete or no question	4	3	7
total number of problems	20	16	36

Second session

Mathematical type	Boys	Girls	Total
A+B static condition	9	3	12
A+B transformation	5	5	10
A-B transformation	10	10	20
A-B static condition	1	1	2

AxB	2	1	3
total single-step problems	27	20	47
A+B+C+D	1	0	1
A-B+C	0	1	1
A+B-C-A	0	1	1
AxB-C	0	1	1
(A+B) x C	1	0	1
A-B - CxD + E	1	0	1
(AxB + CxD) x E	1	0	1
A - BxC & D - ExC	0	1	1
total multi-step problems	4	4	8
hystograms	1	2	3
poorly constructed problems	5	1	6
total number of problems	37	27	64

Third session

Mathematical type	Boys	Girls	Total
A+B static condition	0	2	2
A+B transformation	1	1	2
A-B transformation	3	1	4
A-B static condition	1	0	1
AxB	3	1	4
total of single-step problems	8	5	13
A+B+C	0	1	1
A+B+C+D	0	1	1
A + BxA	1	0	1
(A+B+C) x D	0	1	1
AxB - CxD	1	0	1
(A+B-C) / D	1	0	1
total of multi-step problems	3	3	6
hystograms	1	0	1
poorly constructed problems	2	3	5
total number of problems	14	11	25

All problems in the first set are addition and subtraction problems. This is not surprising since addition and subtraction were the only two operations that the children had been taught at the time. Over half of these problems are in the category of "subtraction with a described transformation". Just prior to the problem writing session the children had been revising addition and subtraction of a two-digit number with a one-digit number. They had not been working with word problems.

In the second set of problems, "subtraction with a described transformation" is still the largest category (about one third of the problems). The operation of multiplication is included for the first time. There are 8 instances of multiple-step problems (13% of all the problems), some of which are very complex.

The problems in the third set seem to be more evenly distributed among the different categories. There are 6 instances of multiple-step problems (24% of all the problems). Just prior to this problem writing session the children had been working on addition and subtraction of two three-digit numbers and on multiplication and division facts.

There is no apparent difference in mathematical type between the problems written by boys and those written by girls.

MATHEMATICAL TYPE : FURTHER ANALYSIS

A second attempt to analyze the mathematical structure of the children's problems was made by using the method of structural analysis developed by Mayer (1982). Mayer analyzed algebra textbook problems and concluded that:

...the relevant information in nearly all of the problems in algebra textbooks could be described using four basic types of propositions:

1. Assignment propositions. This involves giving a single numeric value for some variable. Examples include, "the cost of the candy is \$1.70 per pound", "one pipe can fill a tank in 6 hours", or "the total amount invested was \$4,000".
2. Relation proposition. This involves giving a single numerical relationship between two variables. Examples include, "the length is 2 1/2 times the width", "the area of one rectangle is 64 square inches less than the area of a second rectangle", or "the

rate in still water is 12 mph more than the rate in the current".

3. Question proposition. This involves the question asked in the problem, in which the goal is to find a single numerical value corresponding to a given variable. Examples include, "how much time will it take to empty the tank" or "how many miles will the first car have gone before it is passed?".

4. Relevant fact. This is a fact that is necessary for the integrity of the problem. Examples include the fact that "the same route was used"... or that "the tank is full" in the pipes problem.

In addition, each problem contained information that was not relevant to solving the problem. For example, in the fence problem, "Mr. Zecha", "chain fencing", and "lot" are not directly relevant;..." (p. 201)

Mayer (1982) then used the above categorization to study (a) how subjects (undergraduate psychology students) recalled problems that they had read, and (b) the kind of problems that subjects constructed. He found that relation propositions were harder to remember and occurred rarely in the problems constructed by his subjects: only 3% of all the propositions produced were relation ones.

In this respect, the sample from this study is similar to Mayer's (1982) : out of the 125 problems examined only one contained a relation proposition:

"Anne hade seven candies. Jim hade three times as many. haw many in all?"

The rest of the problems consisted of assignment propositions, question propositions and non relevant facts. A typical problem usually consisted of two assignment propositions and one question proposition. However, there are 19 instances of problems with three or more assignment propositions, four instances of problems with two questions, and nine instances of problems without any question.

Irrelevant fact propositions seem to play a more important role in this sample than in Mayer's (1982). Within this sample, 21 problems contained at least one irrelevant fact proposition; all these 21 problems had been written during the second and third sessions (11 written by boys and 10 by girls). In the 19 cases the irrelevant fact did not involve numerical information, while in two cases it did. Examples:

"My dad went to the store he bought seven pizzas they cost 1.29 each. How much money did he spend?"

"15 kids were playing The kids were winning 8 to 0 5 kids left the game How many kids are left?"

Mayer (1982) does not mention any occurrence of irrelevant fact propositions in the problems constructed by his subjects.

SUMMARY

The typical problem produced by the children in the present study is a single-step problem consisting of three main clauses (two assignment propositions and one question proposition), the most popular type being that of subtraction with a described transformation. A sizeable minority of children, however, constructed problems which were more complex either linguistically or mathematically: elaborate contexts (irrelevant fact propositions), advanced syntactical forms and multiple-step problems.

Several changes are evident in the kinds of problems constructed at the beginning and at the end of the school

year: the problems increase in length as well as in syntactic and mathematical complexity, and they involve larger numbers. However, there seems to be no obvious influence of the introduction of calculators on any of these changes. The size of the numbers - with a few exceptions - is quite consistent with the regular instruction received by the children. In fact, although they had been taught numbers up to about 300 before the second problem writing session, they usually chose one- or two-digit numbers for their problems (95% of the instances in the second set of problems and 79% in the third one).

The results indicate small or no sex-related differences in the length of the problems, in their syntactical structure, in their mathematical content and type, and in the size of the numbers. Boys wrote the shortest and the longest problems and were more likely to produce incomplete sentences; girls had a tendency to use smaller numbers, at least in the first session. There were some differences in the context of the problems produced by girls and boys: while some categories of context, such as inanimate objects, were common to children of both sexes, boys - with one exception - were the only ones to write about violent or conflict situations and about team games, and girls - again with one exception - were the only ones to write about playing and about money outside a buying context. Moreover, when a problem involves human characters, these are overwhelmingly of the same sex as the child who wrote it.

CASE STUDY RESULTS

Case Study A

Child A wrote two problems at the first session, three problems at the second session, and one problem at the third session :

A.1.1 "Bill had 20 apples Todd gave him 20 apples how many apples altogether"

A.1.2 "Tom is 10 years old his brother is 2 years old how many years older is Tom than his brother"

A.2.1 "In Jack's class every child bought two rulers and two Canada copy books. Each book cost 90 cents and each ruler cost 50 cents. How much does it cost for 31 children?"

A.2.2 "Two fishermen were fishing and one fisherman caught 1139 and the other man caught 80 fish. How many more fish did one catch than the other one."

A.2.3 "In the parking lot there are 90 cars on one side and the other side 50. How many cars in all?"

A.3.1 "At the Greenfield parks new library there were 11000 shealfes and on each shealf there were 1000 books and at the old library there were 20 shealvs and on each shealf 100 books. How many more books at the new library than the old one How many more shealfes are there"

Problems A.1.2, A.2.1, A.2.2, and A.3.1 are well above the average length in their respective sets; problem A.3.1 is the longest problem produced in any set. The syntax of problem A.2.1 is remarkable by the fact that the question sentence contains new numerical data: this is very rare in the children's problems.

Except for problem A.1.2, the numbers used tend to be well above the median. This is especially true for the problems written in the second and third session which feature four- and five-digit numbers. Child A produced

three problems of the type "subtraction with a static condition described" (there is only one more instance of a problem of this type in all three sets) and two multiple-step problems, one of them involving two questions. He attempted to solve all of his problems and solved four of them correctly. For problem A.2.1 he gave the answer 86.86\$ instead of 86.80\$ (he does not show how he found it), and he gave 10980 as the answer to the first question of problem A.3.1 (i.e., the difference between the number of shelves instead of the difference between the number of books).

A review of the writing sessions clearly indicates an increase in length, complexity, and numbers' size.

Case Study B

Child B wrote one problem at the first session and one at the third session; she was absent at the second session.

B.1.1 "There was 10 cars in the parking lot 7 cars went away how many cars are left"

B.3.1 "If one can of soup cost 65 cents and you have to get 7 cans how much money do you need?"

The first problem is of a standard type, only slightly above average length and median numbers' size. The last problem had a very interesting and unusual linguistic structure, consisting of a single sentence in hypothetical form. The mathematical type is multiplication, which is also remarkable (there are only 7 multiplication problems altogether). Child B solved both of her problems correctly. The second one involved giving the answer to 7 times 65 cents as 4.55\$.

Case Study C

Child C wrote one problem in the first session and two problems in each of the second and third sessions.

C.1.1 "There was 9 stars in the sky two more came how many stars are there now"

C.2.1 "the boys scored 8 points. The girls scored 6. How many more points did the boys score?"

C.2.2 "Lori-Anne had \$ 1.00 Simon had \$ 2.00. If Lori-Anne bought two apples for 25 cents each what would her change be? If Simon bought 4 apples what would his change be?"

C.3.1 "Anna had 9 candy Jane had 20 candies how many candy all together"

C.3.2 "One horse is \$ 1000. Two horses \$ 2000 20 horses are \$ 20000. How much money for all the horses?"

Problem C.1.1, apart from its original setting, is a rather standard problem, only slightly above average length and median number size. Both problems from the second session are interesting and well constructed. Problem C.2.1 belongs to the "subtraction with a static condition" type. C and A are the only children to have produced problems of this type. Problem C.2.2 is a multiple-step problem, with two questions formulated in hypothetical form.

Problem C.3.1 is below average in length and numbers' size. Problem C.3.2 contains redundant information and an unclear question (which are the horses referred to as "all"?). This may explain why it is the only problem C did not solve correctly.

The quality of C's production is not consistent, but the problems that she wrote at the second session are definitely among the most remarkable ones of the whole sample.

Case Study D

Child D wrote one problem in each of the first and third sessions and four problems in the second session.

D.1.1 "20 kids were playing ball 8 went to play soccer and 5 went to play football how many left playing ball?"

D.2.1 An histogram is presented together with the following questions: "How many pencils are there? How many pappers are there? How many pices of papper and math work is there? How much work books and copy books is there? How many erasers are there?" (The answers to all the questions can be read from the hystogram.)

D.2.2 "If I had 29 apples and I gave one to each kid in my class how many will I have left?"

D.2.3 "I have 6 packs of gum, my friend gave me 5 more. If there are 5 packs of gum in each pack how many will I have?"

D.2.4 "i have 20 coloring books I gave 2 to my friend and 5 to each of my brothers and my aunt gave me 6. how many do I have"

D.3.1 "35 people wanted to play basketball. 13 more people came. 3 went away. then they went into groups of five. How many groups of five are there?"

All of D's problems are of above average length (problem D.1.1 is the longest produced in the first session) and they all have some remarkable features. Four of his six problems are multiple-step, including the one that he wrote at the first session. Problem D.2.1 is a hystogram problem and problem D.2.2 is expressed as a single hypothetical sentence and requires that one piece of information (the

number of the children in the class) be obtained from sources other than the problem statement.

The numbers used by D are not particularly large: they are all one- or two-digit numbers. D correctly solved only two of his six problems. Otherwise the quality of D's production is consistently very high.

4.4 Attitudes

4.4.1 The Attitude Measure

The attitude questionnaire, found in Appendix D, is a set of six questions designed to measure attitude toward problem solving. The format of "happy-sad faces" was modelled after Whitaker's (1980) scale which was shown to be effective with young children. After being developmentally tested with very small groups of children, the questionnaire was piloted in the Grade Three class in June of the preparation year. During the main study the questionnaire was administered as a pretest in September 1983, and a posttest in May 1984.

The test results were not submitted to thorough statistical analysis. The measure was intended only as an indicator of the children's view of problem solving and to assess informally if there had been any change in attitude during the year of using calculators. There were pretest and posttest data for 26 children. In the first five questions, scores of 5 to 1 were assigned to the responses from positive to negative feelings. In question six, scores of 3, 2, and 1 were given to the three choices. The total possible score was 28.

4.4.2 Results

Table 1 shows the number of student responses to each of the six questions in pretest and posttest.

		Responses				
		5	4	3	2	1
Question 1	pretest	19	6	1	0	0
	posttest	22	3	1	0	0
Question 2	pretest	18	8	0	0	0
	posttest	19	4	2	0	1
Question 3	pretest	24	2	0	0	0
	posttest	24	0	1	1	0
Question 4	pretest	12	10	3	1	0
	posttest	14	9	1	1	1
Question 5	pretest	19	3	2	0	2
	posttest	23	1	2	0	0
Question 6	pretest			7	13	6
	posttest			13	13	0

Table 1
Number of Children Giving Each Response

As with many self-report measures used with children, the responses were very positive. The mode for all questions was 5, the most positive response. The question with the greatest positive change between pretest and posttest was Number 5: "Do you like doing math problems with big numbers?" It may be hypothesized that this is due to the use of calculators. Further empirical study is indicated.

Table 2 shows pretest and posttest total scores of each student. A comparison of each child's responses in pretest and posttest shows that even though the initial test was very positive, 14 of the 26 increased in the posttest scores.

Student	Pretest	Posttest	Change
1	27	27	0
2	28	28	0
3	26	24	-2
4	25	27	+2
5	27	28	+1
6	26	28	+2
7	26	28	+2
8	24	27	+3
9	23	23	0
10	26	27	+1
11	25	21	-4
12	24	22	-2
13	25	26	+1
14	28	28	0
15	25	25	0
16	23	25	+2
17	27	28	+1
18	25	28	+3
19	17	13	-4
20	28	28	0
21	27	25	-2
22	25	27	+2
23	21	27	+6
24	21	24	+3
25	25	28	+3
26	27	28	+1

Table 2

Pretest and Posttest Scores of Each Student

These results with third grade students support Whitaker's (1930) statement concerning his fourth grade subjects - that the children possessed a positive attitude toward mathematical problem solving.

4.4.3 Case Study Results

Table 3 presents the results of the four students selected for individual case studies. Subject A gave the most positive responses on both pretest and posttest, scoring 28 on both. Subject B scored 25 in both tests,

indicating no change in attitude. Subject C had scores of 24 and 22, a change of -2. Subject D had the most positive scores on both pretest and posttest.

Case Study	Pretest	Posttest	Change
A	28	28	0
B	25	25	0
C	24	22	-2
D	28	28	0

Table 3

Based on much of the information gathered and summarized in Section 4.2, these results seem to be a good indicator of the self-concept of each child and the attitude exhibited by each in the interview sessions.

CHAPTER 5

CONCLUSIONS

5.1 Limitations of Study

Caution must be exercised in interpreting the results of any study which is limited in scope and number of subjects. But such caution is of even greater importance in a study which is exploratory in nature. The identification of behaviors has been difficult, in spite of the use of both video- and audio-tapes. In many instances the behaviors have been merely inferred since the children rarely exhibited them. Thus the analyses are, in most cases, mere summaries of observations with explanatory comments rather than definite findings.

In order to accommodate the existing tightly-scheduled mathematics program and in order to avoid imposing inappropriate demands on the generosity and willingness of the already busy teacher, certain flexible procedures needed to be adopted for the development of the study. Examples of such procedures that may now be labelled limitations are :

- the extended period spent selecting subjects and subject pairs limited the time period allotted to the actual case studies.
- only minimal in-class time was devoted to the solving of problems and the use of supplementary calculator worksheets.
- relatively minimal instruction and assistance were

provided for the development of the writing of children's original problems.

However, it was important not to give cause to the debilitation of the children's mathematical achievement by decreasing the time available for the regular mathematics program.

Other areas where decisions were made which seemed appropriate at the time but which later produced unexpected results are :

- selection of problems for the interviewing sessions. Although the initial criteria were the appropriateness of the situation for calculator usage and the motivational aspect of the children, it became increasingly apparent that a third factor was of equal importance;
- the attitudinal scale. Certain terms which were used in the scale were possibly interpreted differently by the children and the researchers. Other limitations of this measuring device were noted in Chapter 2;
- pairing of subjects. This became a difficult assignment when such factors as peer pressures and challenges interfered with the desire to please the interviewers.

5.2 General Findings

As indicated by the objectives of this study, the intent of the researchers was to explore how calculators would affect the arithmetical problem solving abilities of

eight-year olds. It was hoped that the study would provide educators with assistance for encouraging children to solve problems creatively, to take advantage of calculators while solving problems, and to construct and solve their own arithmetical problem solving situations.

The features of the study that appeared to be unique were :

- it investigated arithmetical problem solving abilities through the use of calculators with young children (ages 8-9) rather than older ones (ages 11...)
- the duration of the data collection was 1 year rather than the usual short time-block of 4-6 weeks.
- the case study approach, rather than the normal statistical procedure, was used.
- a formal study focussing on the characteristics of children's original word problems (including their constructions and solutions) was undertaken.
- all data collection involved the usage of both video- and audio-equipment and therefore permanent data exists for further investigative studies.

As should have been expected the data from the project was of extensive magnitude and consequently, throughout the study, certain modifications were necessary. These included the major focussing on Objective (1), with limited emphasis on Objectives (2) and (3). As indicated in the report and appendices, a wealth of data was collected: approximately 50 interview sessions (resulting in 90 protocols), and 3

problem writing sessions (resulting in 165 original problems). This resulted in an enormous amount of time being spent on viewing, transcribing, coding, analyzing, graphing, and summarizing the data. It has been difficult to arrive at definite conclusions. A summary of the findings and observations seem to be the most suitable means by which to communicate the results. A list of recommendations will conclude this report.

5.3 Specific Findings

5.3.1 Objective (1) : To observe through a case study approach, the exploratory behavior exhibited by girls and boys when calculators are made available in problem solving situations.

It was apparent that as the children became more proficient at using the calculator throughout the year, it became an effective aid to use while solving problems. There were noticeable changes in the children's uses of the calculator over the period of data collection. Many of these were noted in the discussion of the case study subjects. There was also an evident change in attitude. Earlier verbalizations such as "...when we argue we use calculators..." demonstrated a limited understanding of calculators and thus a lack of interest in it as a tool. But, by the end of the study no such simplistic comments were noted. The change in use reflected the change in effectiveness. As the children became more proficient

in their mathematical skills and their knowledge of the calculator, there was a noted improvement in how effective the calculator was for finding the correct solution. In some cases, the calculator became more than just a computational tool - it became a medium with which to interact and to actually think.

5.3.2 Objective (2) : To investigate the changes that occur in the kinds of arithmetical problems children construct following the introduction of calculators.

Several changes are evident in the kinds of problems children constructed from the beginning to the end of the school year. The problems increase in length as well as syntactic and mathematical complexity. A change is also reflected in the increasing size of the numbers used within the problems. The size of the numbers though, with a few exceptions, is quite consistent with the regular instruction received by children. It cannot be stated for certain whether the calculator had any influence on these changes.

5.3.3 Objective (3) : To observe if any change occurs in children's attitudes towards problem solving as a result of using calculators.

The questionnaire initially indicated that the children had a very positive attitude toward problem solving. This increased slightly in the post-testing session. The question that had the greatest positive increase was the question that asked whether the children enjoyed doing math problems with big numbers. It can only be hypothesized that this may be due to

the children's usage of calculators in the interim period. In order to assume some sense of anonymity regarding the source of the calculators, it was necessary that the children not be specifically questioned by the researchers on their attitude towards calculators.

5.4 Summary of Observations

5.4.1 Problem Solving

Much of the background literature seems to assume that children need extensive instruction in problem solving, but the results of this study reveal that the children were good problem solvers without any structured instruction. It appears from this study that the main requirement for children to be good problem solvers is for them to have the opportunity to solve problems. These problems, of course, should fall within the parameters of the definition of a problem as stated earlier, i.e., 'a situation in which an individual is called upon to perform a task for which there is no apparent algorithm or means to solve it'. It is equally important to note that the problem must be of interest and must challenge the child in order to motivate him/her to attempt a solution. The problems presented to children should consist of a variety of types as well as topics. Certain types of problems seem more appropriate for use with the calculator. In this particular study it was felt that

guess-and-test problems, whether single or multi-stepped, were the most suitable type for calculator usage.

5.4.2 Children's Original Problems

When children have the opportunity to write their own problems, many of the aforementioned conditions of a good problem are fulfilled. At first, children's original problems are sometimes unimaginative or unmotivating, but with practice, this can change. Children often base their writing of problems upon textbook problems which they have encountered. It is when they realize that they encounter problems practically everyday in their school-life and home-life which are worthwhile to write about that children really begin to benefit from this activity. Sometimes the problems are ones they encounter themselves and sometimes they will be problems encountered by family or friends. The child is aware that if the answer is known beforehand, it is not a problem and will therefore challenge him/herself with problems that cannot be immediately solved. Another value of children writing original problems is that the teacher can learn much about the child's interests. This is, in turn, valuable information to know when selecting problems for that child.

5.4.3 Manipulatives and Calculators

Along with the need for good problems, children require manipulatives to use in the problem solving

process. The number and type of manipulatives will depend both on the mathematical and conceptual level of the problem, as well as on the developmental level of the child. In this study it was realized that counters may be of significant value to children at a concrete or semi-concrete level of development and that the child's level of creativeness is the only limitation upon the variety of usages. Case Study Subject C proved this on numerous occasions when she used her counters to represent such things as chickens' and sheep's legs and money. A calculator is another manipulative that should be made available to children during their problem solving sessions. The child is then no longer limited by his/her lack of speed in computation or even a lack of ability to compute some of the operations necessary for solving the problem. However, it is important to note that children require instruction on both the capabilities and limitations of the calculator. It has to be regarded as a tool that can only be effective if the user knows what he/she is inputting and why.

It is obvious, therefore, that teachers, who are often required to adhere to a textbook in order to fulfill provincial or board-level curriculum guidelines, must acquire textbooks whose focus in mathematics is problem solving. As well, the textbook should include calculator instruction which would provide the teacher a means by which to integrate the

calculator into the regular mathematics curriculum.

5.5 Recommendations

Following is a list of recommendations that have been arrived at as a result of this study. It is most interesting that many of these as well as some of the above observations are included in the NCTM (April, 1985) report on the Impact of Computing Technology on School Mathematics.

- 1) In order to become good problem solvers children must be supplied with a wealth of problems of varying types and topics. These problems must be motivating and fall within the realm of the definition of a problem presented in this paper.
- 2) Children should be encouraged to write their own problems for numerous reasons. The children will acquire better writing and communication skills as well as mathematical and problem solving skills, and will be interested in their work because of its relevance to them. Teachers will benefit through the gain of a wealth of information about the children.
- 3) Children must be supplied with manipulative materials during problem solving sessions. The number and types of manipulatives necessary will depend upon the problem. The calculator should be included amongst these manipulatives and proper instruction on its uses and limitations should be given.
- 4) Textbook material should include the recommendations

listed above. Problem solving is to be the focus of mathematics (and, in fact, the focus of the elementary school curriculum in general) and must incorporate the use of calculators and microcomputers into the curriculum.

- 5) Audio-video-equipment should be used by educators and researchers alike in learning about children. A wealth of information can be gathered about children by repeated viewings of these tapes. Allowance is also made for showing the data to others so that children's behavior can be easily discussed.
- 6) The think-aloud technique should be utilized by educators and researchers in learning about children's thinking processes. Children should be interviewed in pairs in the event that a child finds it difficult to verbalize aloud when alone.

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APPENDIX A

PROJECT EQUIPMENT

Panasonic portable videocassette recorder (NV 8420)

Panasonic camera (WV 3230) and case

Sony colour monitor (14")

Bretford model cart, projection table

Star-D tripod and dolly

Sony microphone and mini-phone extension

Video cassettes

Calculators: Texas Instrument, ET model

Menning Dynesen, Executive 2200S model,
GLC/Silver Burdett Pub. Co.

APPENDIX B

Selected samples of Calculator Worksheets

Name _____

40

Calculator Words

Press each number key.

Turn your calculator upside down.

Print the letter you see.

ON	C	0		C	5	_____
	C	1	_____	C	6	_____
	C	2	_____	C	7	_____
	C	3	_____	C	8	_____
	C	4	_____	C	9	_____

Match.

0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

B E G g h I L O S Z

Find each word.

- | | |
|-----------------------|--------------------------|
| 1. Enter 34. _____ | 2. Enter 3507. _____ |
| 3. Enter 335. _____ | 4. Enter 918. _____ |
| 5. Enter 3704. _____ | 6. Enter 771. _____ |
| 7. Enter 7105. _____ | 8. Enter 345. _____ |
| 9. Enter 5508. _____ | 10. Enter 5537. _____ |
| 11. Enter 3215. _____ | 12. Enter 379 919. _____ |

Estimating Sums: 1

Follow these steps.

- Look at the question.

$$\begin{array}{r} 37 \\ + 51 \\ \hline \end{array}$$

- Round and add.

Think
 $40 + 50$

- Record the estimate.

Estimate
 90

- Find the actual sum.



Sum
 88

		Estimate	Sum
1.	$\begin{array}{r} 64 \\ + 18 \\ \hline \end{array}$		
2.	$\begin{array}{r} 28 \\ + 39 \\ \hline \end{array}$		
3.	$\begin{array}{r} 43 \\ + 48 \\ \hline \end{array}$		
	$\begin{array}{r} 19 \\ + 77 \\ \hline \end{array}$		
5.	$\begin{array}{r} 32 \\ + 61 \\ \hline \end{array}$		
6.	$\begin{array}{r} 51 \\ + 29 \\ \hline \end{array}$		
7.	$\begin{array}{r} 34 \\ + 15 \\ \hline \end{array}$		
8.	$\begin{array}{r} 74 \\ + 17 \\ \hline \end{array}$		
9.	$\begin{array}{r} 63 \\ + 31 \\ \hline \end{array}$		
10.	$\begin{array}{r} 28 \\ + 19 \\ \hline \end{array}$		
11.	$\begin{array}{r} 34 \\ + 34 \\ \hline \end{array}$		
12.	$\begin{array}{r} 50 \\ + 16 \\ \hline \end{array}$		

		Estimate	Sum
13.	$\begin{array}{r} 39 \\ + 52 \\ \hline \end{array}$		
14.	$\begin{array}{r} 57 \\ + 26 \\ \hline \end{array}$		
15.	$\begin{array}{r} 37 \\ + 32 \\ \hline \end{array}$		
16.	$\begin{array}{r} 49 \\ + 38 \\ \hline \end{array}$		
17.	$\begin{array}{r} 56 \\ + 22 \\ \hline \end{array}$		
18.	$\begin{array}{r} 72 \\ + 19 \\ \hline \end{array}$		
19.	$\begin{array}{r} 29 \\ + 58 \\ \hline \end{array}$		
20.	$\begin{array}{r} 48 \\ + 48 \\ \hline \end{array}$		
21.	$\begin{array}{r} 49 \\ + 47 \\ \hline \end{array}$		
22.	$\begin{array}{r} 36 \\ + 21 \\ \hline \end{array}$		
23.	$\begin{array}{r} 83 \\ + 18 \\ \hline \end{array}$		
24.	$\begin{array}{r} 29 \\ + 31 \\ \hline \end{array}$		

Hit the Target!

Follow these steps.

- Look at the target product.
- Circle 2 numbers which will give a product closest to the target number.
-  Find the product of the 2 numbers.
- Find out if there was a better choice. If there was, write it in the last column.

	Target Product	Circle 2 numbers.	Product of 2 Numbers	Was there a better choice?
1.	100	21 9 13 5		
2.	185	32 7 30 6		
3.	300	8 7 41 10 35		
4.	550	20 50 21 30 11		
5.	96	12 11 9 13 7		
6.	150	31 41 5 7		
7.	100	40 2 16 6 5		
8.	200	41 7 5 35 6		
9.	150	8 18 3 4 61		

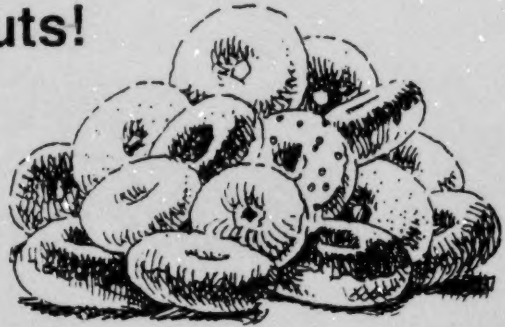
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Doughnuts!

Follow these steps.

- Read the shopping list.
- Fill in the cost of 1 doughnut.
- Estimate the cost of the number of doughnuts.

-  Find the actual cost of the doughnuts.



Jelly
27¢



Cake
19¢



Chocolate
38¢



Honey
31¢



Coconut
32¢



Nut dip
32¢



Cinnamon
29¢

	Shopping List	Cost of 1 doughnut	Estimate	Cost
1.	7 cake doughnuts			
2.	2 chocolate doughnuts			
3.	3 jelly doughnuts			
4.	5 coconut doughnuts			
5.	7 honey doughnuts			
6.	4 nut dip doughnuts			
7.	6 cinnamon doughnuts			
8.	9 cake doughnuts			
9.	7 jelly doughnuts			
10.	9 cinnamon doughnuts			
11.	6 chocolate doughnuts			
12.	8 cake doughnuts			
13.	6 coconut doughnuts			
14.	5 honey doughnuts			
15.	7 nut dip doughnuts			

Estimating Sums: 4

Follow these steps.

- Look at the question.

- Round and estimate.

- Loop the number closest to your estimate.



- Find the actual sum.

$$173 + 201 + 515$$

Think

$$200 + 200 + 500$$

$$800 \text{ (900) } 1000$$

$$\text{Sum} \\ 889$$

		Loop your estimate.	Sum
1.	273 + 106 + 443	700 800 900	
2.	917 + 188 + 293	1400 1500 1600	
3.	406 + 713 + 522	1600 1700 1800	
4.	673 + 792 + 886	2200 2300 2400	
5.	493 + 215 + 742	1300 1400 1500	
6.	217 + 507 + 101	800 900 1000	
7.	496 + 231 + 270	800 900 1000	
8.	742 + 139 + 474	1200 1300 1400	
9.	903 + 669 + 235	1800 1900 2000	
10.	820 + 195 + 386	1300 1400 1500	
11.	158 + 219 + 880	1300 1400 1500	
12.	493 + 297 + 740	1300 1400 1500	
13.	803 + 159 + 275	1300 1400 1500	
14.	315 + 851 + 158	1300 1400 1500	
15.	476 + 671 + 216	1200 1300 1400	
16.	395 + 821 + 405	1600 1700 1800	
17.	862 + 217 + 159	1200 1300 1400	
18.	495 + 375 + 236	1100 1200 1300	
19.	819 + 428 + 215	1300 1400 1500	
20.	679 + 496 + 183	1300 1400 1500	
21.	795 + 328 + 804	1700 1800 1900	
22.	333 + 444 + 555	1200 1300 1400	

Estimating Differences: 1

Follow these steps.

- Look at the question.

- Round and subtract.

- Record the estimate.

- Find the actual difference.



$$\begin{array}{r} 59 \\ - 21 \\ \hline \end{array}$$

Think
 $60 - 20$

Estimate
40

Difference
38

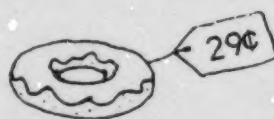
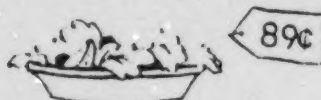
		Estimate	Difference
1.	$\begin{array}{r} 72 \\ - 19 \\ \hline \end{array}$		
2.	$\begin{array}{r} 59 \\ - 31 \\ \hline \end{array}$		
3.	$\begin{array}{r} 86 \\ - 14 \\ \hline \end{array}$		
4.	$\begin{array}{r} 57 \\ - 16 \\ \hline \end{array}$		
5.	$\begin{array}{r} 81 \\ - 48 \\ \hline \end{array}$		
6.	$\begin{array}{r} 73 \\ - 41 \\ \hline \end{array}$		
7.	$\begin{array}{r} 56 \\ - 22 \\ \hline \end{array}$		
8.	$\begin{array}{r} 89 \\ - 38 \\ \hline \end{array}$		
9.	$\begin{array}{r} 74 \\ - 21 \\ \hline \end{array}$		
10.	$\begin{array}{r} 29 \\ - 18 \\ \hline \end{array}$		
11.	$\begin{array}{r} 96 \\ - 24 \\ \hline \end{array}$		
12.	$\begin{array}{r} 87 \\ - 42 \\ \hline \end{array}$		

		Estimate	Difference
13.	$\begin{array}{r} 68 \\ - 14 \\ \hline \end{array}$		
14.	$\begin{array}{r} 72 \\ - 59 \\ \hline \end{array}$		
15.	$\begin{array}{r} 36 \\ - 28 \\ \hline \end{array}$		
16.	$\begin{array}{r} 67 \\ - 23 \\ \hline \end{array}$		
17.	$\begin{array}{r} 69 \\ - 37 \\ \hline \end{array}$		
18.	$\begin{array}{r} 83 \\ - 69 \\ \hline \end{array}$		
19.	$\begin{array}{r} 87 \\ - 23 \\ \hline \end{array}$		
20.	$\begin{array}{r} 88 \\ - 79 \\ \hline \end{array}$		
21.	$\begin{array}{r} 69 \\ - 33 \\ \hline \end{array}$		
22.	$\begin{array}{r} 75 \\ - 27 \\ \hline \end{array}$		
23.	$\begin{array}{r} 52 \\ - 25 \\ \hline \end{array}$		
24.	$\begin{array}{r} 73 \\ - 46 \\ \hline \end{array}$		

Name _____

36

Lunch Math



Find the cost of each lunch.

Toni's

1 hamburger _____

1 salad _____

1 milk _____

Total _____

Barney's

2 hamburgers _____

1 milk _____

1 muffin _____

Total _____

The Gills'

3 hotdogs _____

2 salads _____

3 milks _____

3 muffins _____

2 apples _____

1 orange _____

Total _____

The Kovacs'

3 hamburgers _____

2 hotdogs _____

4 salads _____

2 milks _____

3 juices _____

4 muffins _____

1 apple _____

Total _____

APPENDIX C

Problems solved by children

Appendix C

PROBLEMS

WUZZLE

You are going to make some Wuzzles to sell at the Spring Fair. Yarn for one Wuzzle costs 10¢. Eyes cost 5¢ each. Pipecleaners cost 3¢ each. The sponge foot costs 3¢ each. How many Wuzzles can you make for \$5.00? (How much would you sell them for? How much extra money would you make?)

NEWSPAPER

Julie delivers newspapers from Monday to Saturday. On Monday she delivers 27 newspapers. If each day she delivers 10 more newspapers than the day before, how many newspapers does she deliver on Saturday? (How many does she deliver in a whole week?)

BOOK

Open the book so that the sum of the pages is 89. 155.

PROBLEMS

SIGNS

Put +, -, X, $\div$ in the boxes to make the number sentence true.

$$64 \square 8 \square 5 = 3$$

$$37 \square 8 \square 166 = 130$$

FARMYARD

In the Farmyard there are 18 animals. Some are chicken and the rest are sheep. If there are 58 (46) legs altogether, how many of the animals are chickens and how many are sheep?

CARNIVAL

You are going to the carnival together.

You have 30 tickets to share.

Which rides will you go on?

Keep track of how you use your tickets.

Train Ride costs 3 tickets

The Ferris Wheel costs 2 tickets

The Tilt-a-Rock costs 3 tickets

The Drive a Dune Buggy is 4 tickets.

MAP (provided)

If you must make deliveries from Greenfield Park to Mirabel, Dorval, and Ormstown, what is the shortest way to make all the stops and return to Green Park? You may make the stops in any order.

BEANBAG TOSS

Beanbag Toss: In this game you score points by tossing beanbags into the holes. How can you score exactly 100 points by tossing 3 beanbags?

MENU

You are going to St. Hubert's for your dinner. You have \$15.00 to spend between the two of you. Write down what you are going to buy, how much it costs, and how much change you will have.

SHAMROCKS

You are delivering the shamrocks on St. Patrick's Day to the houses on Miller St. At the Adams house you deliver 15 shamrocks. At the Browns' you deliver two times as many as you delivered to the first house. At each house you deliver two times as many as you did to the house before. At which house would you deliver 120 shamrocks?

WHEEL

Problem: Arrange the chips on the wheel so that when you multiply the three numbers on each line, the answer is 360. One line is completed already.

3 6 9 20 30 60

NUMBERS

I

What numbers belong in the empty circles? (Each problem on a separate card)

(a) $\begin{array}{r} 86 \\ 40 \\ \hline 109 \end{array}$

(b) $\begin{array}{r} \bigcirc 25 \\ 64\bigcirc \\ 1\bigcirc 8 \\ \hline 1475 \end{array}$

(c) $\begin{array}{r} 86\bigcirc \\ 9\bigcirc 1 \\ 037 \\ \hline 2\bigcirc 92 \end{array}$

II

There are two numbers written on the other side of this card.

When you add the numbers the sum is 23.

When you multiply the numbers the product is 112.

What are the two numbers?

APPENDIX D

Attitudinal Measure

NAME _____

Circle the face or the word that best gives your answer to each question.

1. Do you like to solve problems in math?



2. Do you like looking for number patterns?



3. Do you like doing math puzzles?



4. Is solving math problems easy for you?



5. Do you like doing math problems with big numbers?



6. Do you solve math problems in your spare time?

often

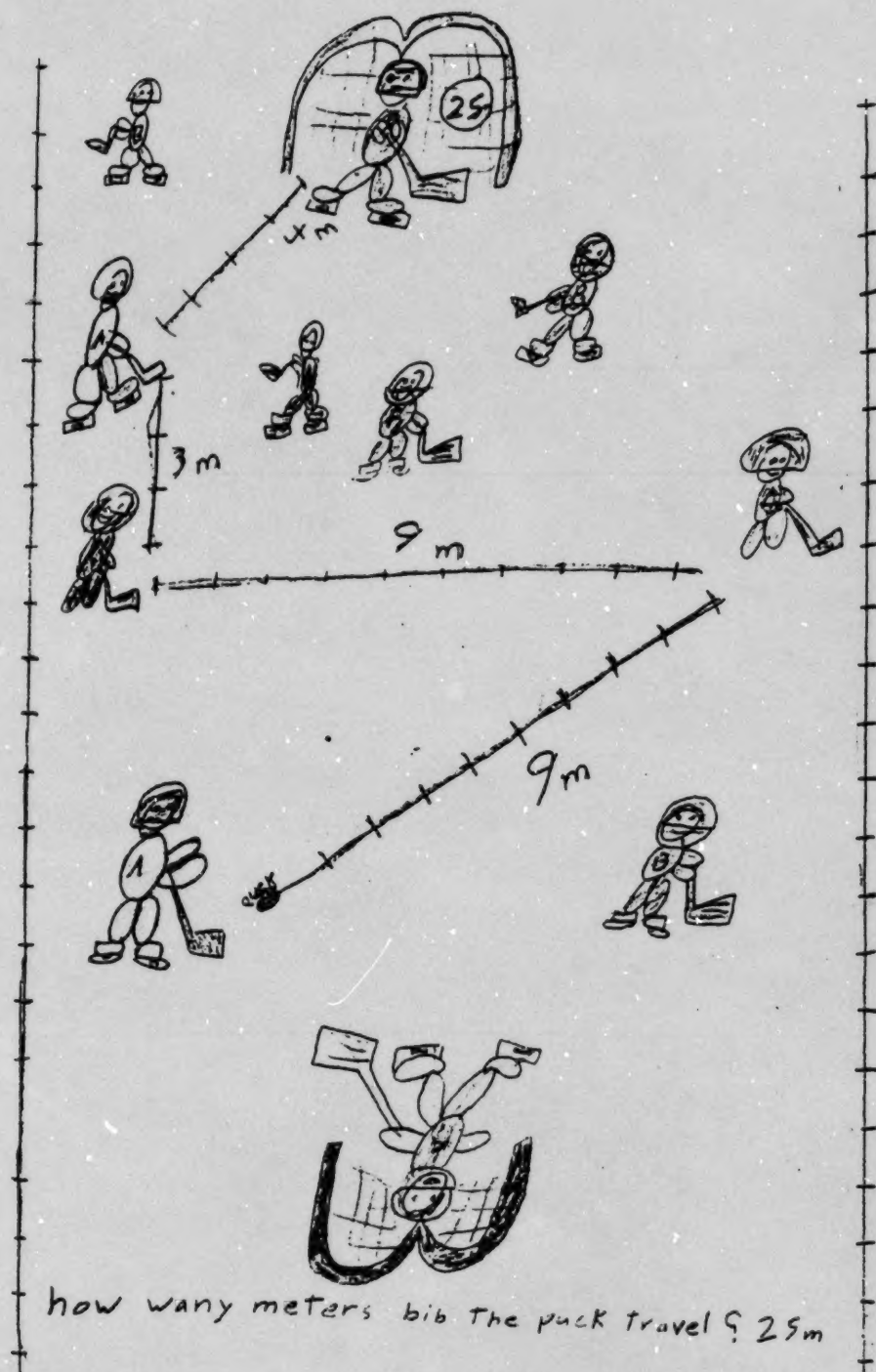
sometimes

never

DATA (WHEN?)	MEDIA (HOW?)	PROCEDURES (WHY?)	SKILLS (WHY?)	OUTCOME
DATE?	C - CALCULATOR	CO - COUNTING	CH - CHECKING	CSO - CORRECT SOLUTION
PROBLEM?	(CREF - CALCULATOR REFERENCE)	A - ALGORITHM	E/G - ESTIMATING	ISO - INCORRECT SOLUTION
PROBLEM TYPE?			E/G - GUESSING	
ALONE OR ACCOMPANIED BY?	P - PAPER & PENCIL		E/G - ESTIMATING &/OR GUESSING	CPSO - CORRECT PARTIAL SOLUTION
	CO - COUNTERS, ILLUSTRATIONS PICTURES			IPSO - INCORRECT PARTIAL SOLUTION
INTERVIEW CODING	<u>P. CATEGORIES</u>			
I - INSTRUCTION	PL - PAPER LIST			
I* - INTERVENTION RESPONSE	Pd - PAPER DIAGRAM			
I* - INTERVENTION INITIATED	Pt - PAPER TALLIES			
	Pc - PAPER CHART			
	PREC - PAPER FOR RECORDING			

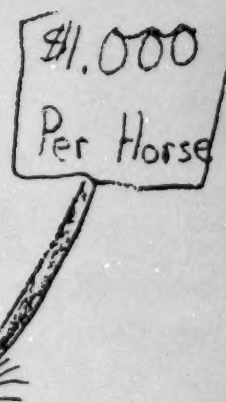
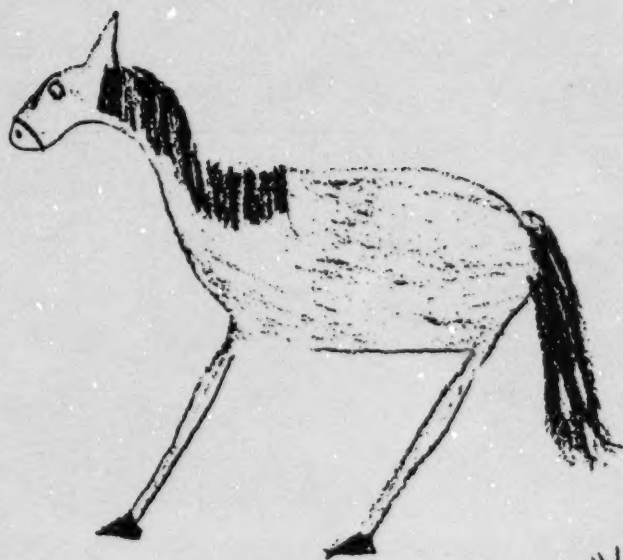
APPENDIX F

Children's original word problems



One horse is \$1,000. Two horses
\$2,000. 20 horses are \$20,000.

How much money for all the horses?



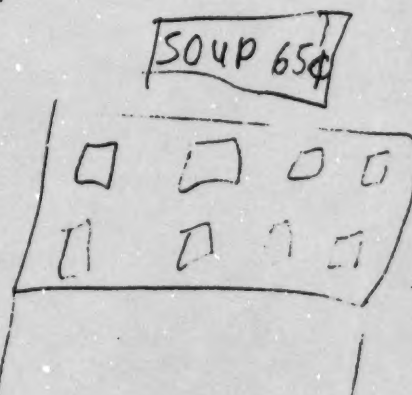
If one can of soup cost 65¢
and you have to get 7 -
cans how much money do
you need?

Answer. $7 \times 65 = \$4.55$

1.



2.



At the Greenfield park's new
 library there were 11000
 shelves and on each shelf
 there were 1000 books and at
 the old library there were 20
 shelves and on each shelf
 100 books. How many more
 books at the new library
 than the old one
 How many more shelves are there.

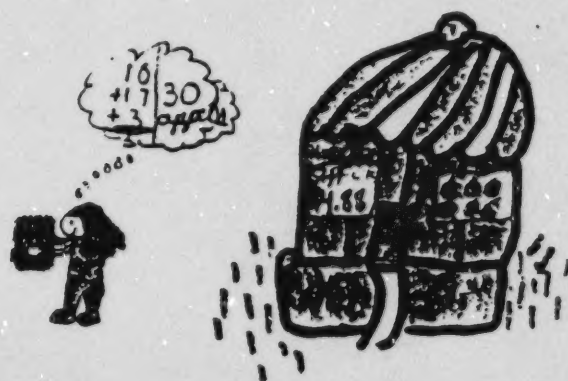
There are 10780 shelves more.

There 900 more books.

1.

Maria had ten apples she bought seven more
and then she went back yesterday
and after her lunch she bought 3

$$\begin{array}{r} 10 \\ + 17 \\ + 3 \\ \hline 30 \end{array}$$

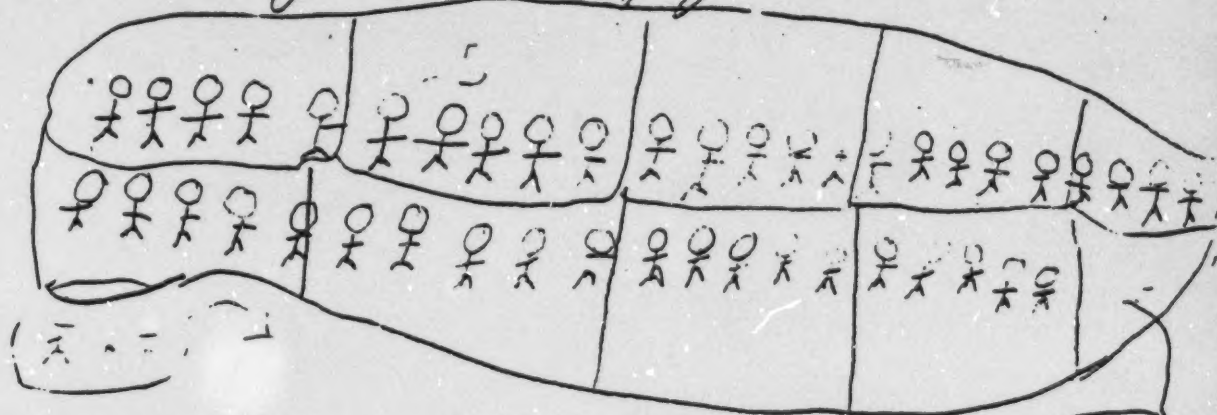


BEST COPY AVAILABLE

BEST COPY AVAILABLE

35 people wanted to play basket-
ball. 13 more people came. 3 went away.

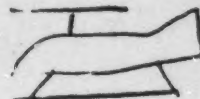
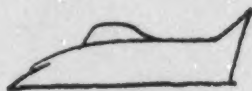
Then they went into groups of five
How many groups of five are there?



ANS. 45 people

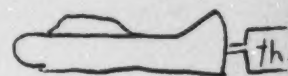
are in groups of 5 = $9 \times 5 = 45$

71 People went to see the plane show it was
3 dollars to get in. how much money did
they collect.



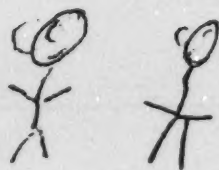
Plane Show

1017891

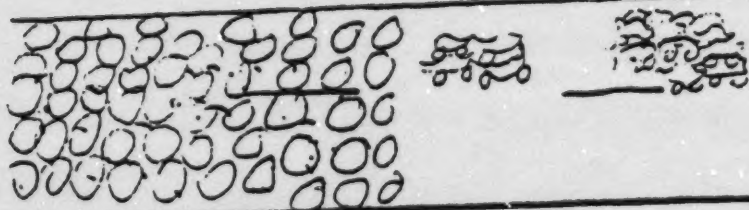


I went to the pet shop, and I
 went in and I wanted some snake
 and a snake cost \$30.00.00 and
 I want 20 snakes how
 much will it ^{answer} cost.

do a picker



There were 12 cars
on the street then 4
cars came along.
Then 54 cars came
along there were
3 people in each
car. How many
cars and people altogether
there are 280 in all



Kathie had 15¢. On the way to the store she found 35¢. she walked again and she found 65¢. And then she found 1.00. How much did she have now

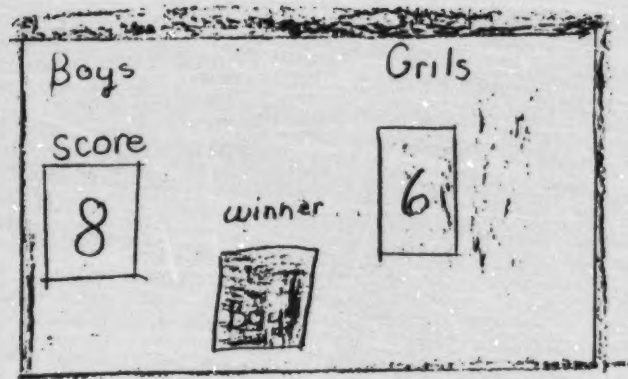
2.15



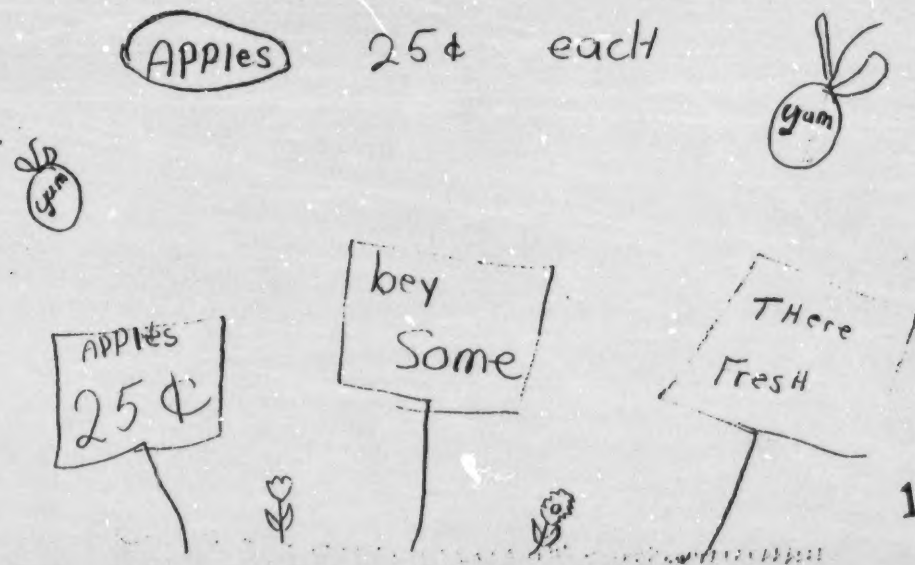
U

The Boys scored 8 points. The girls scored 6. How many more points did the Boys score?

Answer: 2 more points



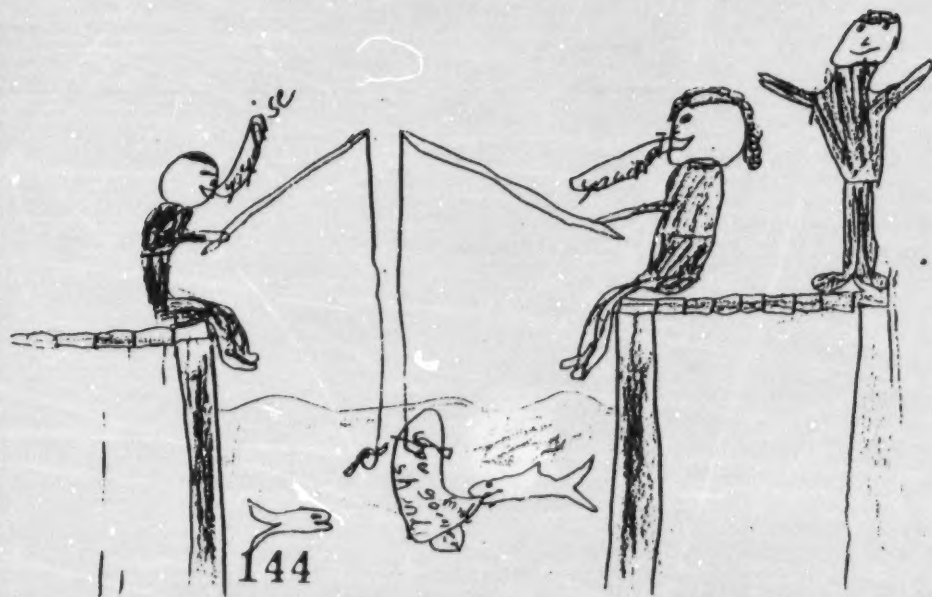
Q Lori-Anne had \$1.00. Simon had \$2.00. If Lori-Anne bought two apples for 25¢ each, what would her change be? Her change would be 50¢. If Simon bought 4 apples, what would his change be? \$1.00



In The war: There were 4 boats
and 5 air planes, how many were there all T



There were 4 girls, Michael J Kissed 3 how many
didn't get kissed. $4-3=$



2 fishermen were fishing 1 caught 35 the
another caught 12 how many were there all together

$$\begin{array}{r} 35 \\ + 12 \\ \hline \end{array}$$

APPENDIX G

Protocols

212414VA Y900 T238

Book Problem
October 20, 1983

St. Jude School

L - Subject B
M - Subject D
I - Interviewer (Jeannine)
I(2) - Interviewer (Denise)

Problem

Time 19:02

- Ins :
1. I: Lisa, did you get the problem? Do you understand the problem?
 2. L: I didn't read it all.
 3. I: First, open the book.
 4. L: What are the two numbers at the bottom of the pages? What...
 5. I: OK. Wait a minute. Answer that question.
 6. L: Oh, you hafta find two numbers that add up to 153?
 7. I: Open the book. Anywhere.
 8. M: Anywhere in the book?
 9. I: Stop. OK. Now what does it say? What are the two numbers at the bottom of the pages?
 10. L: 14, 15
 11. M: 14, 15. Oh!
 12. I: What is the next question?
 13. L: What do you get when you add these two numbers?
 14. M: 14 and 15, 29.
 15. L: 29.
 16. M: Is it 153? No! Somewhere in the hundreds.
 17. L: You'll hafta find... no, don't go to the hundreds. You hafta find numbers that
- Ca Ce/g

- add up to 153.
 Cref Can we do this with our calculators?
 Just put 153 minus...
- Ca Ce/g 18. M: (whispering - unintelligible)
19. I: Don't whisper. Talk out loud so we can hear you. We want to hear what you're thinking.
20. M: Way too over.
21. L: Um hum.
- Ce/g Ca 22. M: 153 minus... It equals zero so we gotta find another one, lower, way lower. Hey! I know it! 76 plus seventy...
23. L: No.
- Ca Ce/g 24. M: Seventy...
- e/g 25. L: 74 and 75.
26. I: Michael, tell us what you're doing on your calculator.
27. L: 149. No, nope, it's not enough. 149.
28. M: How about this one?
- e/g 29. L: Nope, not it either. Maybe around 81.
- Ca Ce/g 30. M: 80.
31. L: 160 plus... (doing mental arithmetic)
32. M: ...plus 81 equals...
- e/g 33. L: 161. Nope, too high. Right in here. 78, 79.
- Ca Ce/g 34. M: 78 plus 79 equals...
35. L: 120.
- Ce/g Ca 36. M: No, so it can't be that. 76. 76 plus 77...
- Ca Ccso 37. L: Yep! 76 and 77.
- Ccso 38. M: Yep.
39. I(2): Well, that was quick!

40. I: You got it!
41. L: Take 153 minus the number (ouch)
42. M: 153 minus 77 equals 76.
43. L: So that you know that's the answer.
44. M: So you know that's the answer.
45. L: ...or you can put 76 and 77.
46. I(2): My, that's like the kind of problem in your math class the last time. That's the type of thing you were doing. You were doing if you knew 5 plus 7 was 12, then you knew that 7 plus 5 was 12 and you knew that 12...
47. L: 12 minus 5 is 7.
48. I(2): ...and 12...
49. L: 12 minus 7 is 5.
50. I(2): So in your math class...
51. M: Maybe we can try 250? ...or something that would equal 255?
52. L: 250 is easy. It's just 125 and 125 is 250. No, it can't be that - it has to be an odd number, it has to be an odd number.
53. M: Or how about...
54. L: It can't be.
55. I: Ah, here's a good question for you. Will any odd number work?
56. L: Yes, but not an even number, cause that's one even number and one odd number.
57. I: That's very good! So it has to be an odd number, but will any odd number work, do you think?
58. M: No, not every odd number.
59. L: Not every one.
60. I: How can you find out?

61. L: Well, if you have a book with 2000 pages...
62. I: Well, for example, take the number 9. That's easy.
63. L: Yeah. 4 and 5.
64. M: Yeah. 4 and 5.
65. I: Take the number 17.
66. M: 17... 7 plus 10.
67. L: 6 plus 5.
68. M: 17! 6 plus 5? 10 plus 7.
69. L: Not 10 plus 7. Cause you don't see 10 and then 7, or you don't see 7 then 10.
70. M: ...or it could be 8 plus...
71. L: 8 plus um, um, um, 8 plus 8 equals 16.
72. M: 8 plus 9
73. L: 8 plus 9
74. M: It could be that or 10 plus 7.
75. L: It can't be 10 plus 7 cause you don't see 10 and then 7.
76. M: ...or 1 plus 12.
77. L: 1 plus 12 is 22.
78. I(2): What about 59?
79. M: 59? That could be 25 plus 34.
80. L: What?
81. M: 20 plus 30, 20 plus 30 equals 50.
82. L: What did you say? 57?
83. I(2): 59.
84. L: 59.
85. M: 20 plus 30, 50... 50 plus 5 equals... oh, 34 plus 25... 4 plus 5 equals 9.

86. L: I can't think! How can you say 34 and 25? Cause 25 and 34... It would hafta be 24 and 25. It can't be 34 and 25 cause they don't come together in pages.
87. I: Try to find the two pages when you open the book to get 35.
88. M: 35, 35.
89. L: That'll be, no.
90. M: 35.
91. L: 17.
92. M: 17, 18
93. L: 16 and 17, uh 17 and 18, no, 16 and 17.
94. I: 16 and 17? What's the sum of 16 and 17?
95. L: That's 23.
96. I: What is it?
97. L: 16 and 17 is 23.
98. M: No, 33. So 18 plus 19, 37. Which number did you say?
99. I: 35.
100. L: 35 minus uh 18, 17 and 18.
101. M: 35.
102. I: So you can't open the book and have the sum of the two pages 35.
103. M: Unless you're going like this.
104. L: But I said, but I said, 17 and 18, then I said no that can't be. So then I put down 16 and 17.
105. I: That is true. 17 plus 18 is 35, but when you open the book you don't see 17 and 18 together, do you?
106. L&M: No.
107. I: That's good.

Newspaper Problem

November 24, 1983

St. Jude School

M - Subject D

L - Subject B

I - Interviewer - Jeannine

I(2) - Interviewer - Denise

Problem

Julie delivers newspapers from Monday to Saturday. She delivers 27 newspapers on Monday. If each day she delivers 10 more newspapers than she did the day before, how many newspapers does she deliver on Saturday?

Time 20:10

iso 1. L: 37

iso 2. M: 37...no, 10.

3. L: No, wait, 10 more... that's 30.

4. M: Wait, there's 7 days in a week, but she doesn't go on Sunday, or Saturday...
(whispers)

iso 5. L: 97.

iso 6. M: 97?

(Both look at interviewer.)

7. I: Don't look at me. You decide what you think it is.

(take markers and start writing - microphone is fixed)

Pl Piso 8. M: Testing 1, 2, 3... (laughs) Uhm... $x 10$
 $x 10 x \dots$ wait there's 6 days. $6 x \dots =$
 $\dots 87$.

Prec 9. L: Just a minute...97.

Pl 10. M: $\dots + \dots$ uh (whispers)Prec 11. L: look. $27 + 7 \text{ more } 10\text{'s} = 27 x 2$ (writes)

12. M: 6... 10's (whispers)

13. L: No. 97, 'cause look, $27 + 70$ is 97.

Pa Pl 14. M: OK... ah (whispers)

15. I: Talk out loud and tell us what you're doing.

(whispering and writing)

Piso

16. M: 87. That's the answer. 87.

17. L: 97. 97 'cause there's 7 days in a week and if she... (inaudible)

18. M: But she doesn't go on Sunday... from Monday to Saturday.

iso

19. L: So it's only 67 then.

20. M: 87.

21. L: ...'cause there's not 8 days in a week. There's only 7. Take away that 1.

22. M: 6 ...look... 10...

23. L: That makes no sense.

24. M: She delivers 10 more on each day... 1, 2, 3, 4, 5, 6, plus the 2 that she had, so it's 87.

25. L: 67

26. M: 87

27. L: There's not 8 days in a week, Michael. How many...

28. M: She had 27 newspapers already.

29. L: I know.

30. M: ...and then...each day she gets newspapers.

31. L: 10 extra ones...

32. M: 10 extra...

33. L: So then 27... 6 more days.

Pd

34. M: ...and then there's 37, then 47, 57, 67, 57, 47 (draws lines on paper - Lisa rereads problem) ...see, it all equals...

Cref

35. L: That makes no sense. Put that in your

calculator and it'll go berserk... makes no sense.

- Ca 36. L: You put + 0 + 0 30, 49, 50, 60 ,70, 80, 89... + 10 + 10 +10...
- Ca 37. M: 27...27 + 10 + 10 + (whispers while using the calculator)
- Cref 87, even says in the calculator.
38. L: You said, how much did you say it... you said it was 77?
39. M: I said it was 87.
- iso 40. L: Yeah, it is 87. I thought you said 77. Yeah, it's 87.
41. M: That's ...
42. I: OK. Explain to me how you figured that out.
43. L: Well, 27... there's 27 newspapers. 10 more in each day, so she goes 6 days and then 6 more plus 27 is 87.
44. I: Alright. Could you put that in a chart for me?
45. M: Chart? (Michael turns paper over.)
46. L: What are you doing? Michael!
- Pl Pa 47. M: That's yours. (gives her marker)
48. L: Michael, I don't have any room to write mine!
49. I: He can write and you can tell him what to write. Now what does each one of these 10's mean, Michael?
50. M: You're adding on a 10.
51. I: Why are you adding on a 10?
52. M: Because each day she gets...
53. L: ...extra...
54. M: She gets extra 10 more, and then by the end of the week she has sold 85 newspapers.

55. L: 85...87!
56. M: 87... 1, 2, 3, ... (whispers and writes)
57. I: And what does that mean? How many does she deliver on Tuesday?
58. M: Tuesday she delivers ...37.
59. I: and...
60. M&L: ...then 47 and 57 and 67 and 77 and 87
- Pa 61. M: Oops!
- Proc 62. L: Oops! Michael, move over. Do it there. I'll do mine here, 'cause it's easier.
63. M: + 37... (whispers, writes)
64. L: Not + 37, 27 + 37... She didn't say we had 37 more!
65. M: 27 (whispers) What are you putting down?
66. L: Not + 37.
- Int • 67. I: OK. Maybe it would help to read the problem once more.
68. M: Julie delivers newspapers from Monday to Saturday. On Monday she delivers 27 newspapers. If each day she delivers 10 more newspapers than the day before, how many newspapers does she deliver on Saturday? She delivers...
69. L: 37... oops!
- e/g 70. M: A hundred and ... lower?
71. I: You were on the right track.
72. I(2): What could you put beside your numbers...
73. M: 87.
- Int • 74. I(2): ..to keep track? What are you forgetting to keep track of? You're keeping track of numbers... What else can you keep track of? What is it you're trying to find out?

75. M&L: How many she delivers on Saturday.
- e/g 76. M: ...trying to find out that... but we're trying to ...ah, discover, ah,... I don't know. Is it... is it multiplication?
- Ca 77. L: Oh, I got it now.
78. I: Lisa has it.
79. M: What?
- Ca 80. L: Well, I'll show you (using calculator, doesn't show Michael)
- e/g iso 81. M: 97, I think.
82. L: I did a little mistake...OK.
83. M: $27 \times \dots$ (whispers)
- Pl 84. L: Just a second. I'm ... (whispers) ...27
85. M: $16 + \dots + 67$ (whispers) -17...
86. I: Talk out loud and tell us what you're thinking.
- Piso 87. L: I think it's 77... ah 87, because if she has... if she delivers 27 on... if each day she delivers 10 more... so she delivers 37 then 47 then 57...uh?
88. I: When does she deliver 57?
89. M: On S...
90. L: Ah, 27 on Monday, eh?
- Int • 91. I: OK. why don't you put that down so you'll know how many she delivers on Monday.
- Pc 92. L: 27... and then 37. I'll put...
- Prec 93. M:
94. I: That's a good idea.
95. I(2): That's what you were forgetting.
96. M: Monday, Tuesday, Wednesday... On Thursday she does 57.
97. L: Yeah.

98. M: Friday...67.

Peso 99. M&L: (shout) It's 77!

100. I: (laughs) And what trouble were you having?

101. I(2): What were you forgetting to keep track of?

102. M&L: How many she delivers on the first day.

103. I(2): No, you remembered 27, but what were you forgetting?

104. L: To put the days on...

105. M: To put the days on.

106. L: So we were both wrong. (to Michael)

107. M: Yeah.

Time 31:08

Newspaper Problem

December 1, 1983

St. Jude School

L - Subject C

S - Simon

I - Interviewer (Denise)

Problem

Julie delivers newspapers from Monday to Saturday. She delivers 27 newspapers on Monday. If each day she delivers 10 more newspapers than she did the day before, how many newspapers does she deliver on Saturday?

Time 33:41

1. L: What is this talking about?
2. I: This one you've got to read slowly.
3. S: Oh, I get it. Look, see if she delivers 27 newspapers on Monday to Saturday, how many more newspapers... how many days would she take to deliver 10 more? Do you get it now?
4. L: Uh-huh. She delivers...
5. S: How many more days...
6. L: 4. No.
7. S: How many each day...
8. I: I'd read it very carefully again to yourself.
9. S: Am I right about that?
10. I: You'll see. You read it one more time to yourself. I think Lori better read it too. Read it carefully.
11. L: Oh, now I understand. You gotta figure out how many, how many...
12. S: newspapers
13. L: how many more, how many days it would take her to deliver 10 more. That would be 37 newspapers.

e/g

- Int •
14. I: What's the last question ask you?
15. L: How many newspapers does she deliver on Saturday?
16. S: This is hard.
17. L: This is very hard.
18. I: Tell me what you know first.
19. S: I know $2 + 2 = 4$.
20. L: You know that she delivers 27 newspapers, on Monday she delivers 27 newspapers, and if each day she delivers 10 more newspapers than the day before, how many newspapers does she deliver on Saturday?
21. S: Lori
22. L: I don't know how many she delivers on Saturday.
23. S: Lori
24. L: Yeah
25. S: It took a week just to get 27 newspapers.
26. L: OK. She delivers 27 newspapers on Saturday. How many will she deliver on...
27. S: Monday to Saturday.
28. L: Monday to Saturday? Oh.
29. S: says that Saturday... on Monday she... Oh, on Monday she delivers... so...
30. L: I know that. How many does she deliver on Saturday?
31. S: If she delivers 27 newspapers in one day, she can deliver 10 more.
32. L: No she couldn't - she'd be pooped out.
33. S: No, I mean the next day.
- Int •
34. I: OK. How many does she deliver on Monday, Simon?
35. S: She delivered, uh, 27.

36. I: and what's this say, Lori?
37. L: Each day she delivers 10 more newspapers that the day before, how many...
- Int • 38. I: OK. So on Monday she delivers
39. S: 10
40. I: On Monday she delivers...
41. S: Oh, 27.
- Int • 42. I: How many does she deliver Tuesday, Lor?
43. L: 37?
44. I: How did you get that?
45. L: Um, cause 10 more would be 30 and you still got the ones.
46. I: OK. So how many does she deliver on Wednesday?
47. S: Uh, 47.
48. I: Oh.
49. L: ...then 57.
- Int • 50. I: How could you write some of this down to how me? I want to know Saturday. You know Monday, you know how to figure out Tuesday...
51. S: I got an idea! Can I use pencil?
52. I: I'd rather if you use marker cause the camera can't make out the colour. OK now.
- Pl 53. S: Oh great! You gave me my favourite colour.
- Pl 54. L: 67 on Thursday, Friday 77.
55. I: What's the easiest way to write this down?
56. S: Changing the ones.
57. L: Count by tens.

- Int • 58. I: How can you figure out which day is which? First you write down what you know. You write down...
59. L: I know there's 27 newspapers.
60. I: on...
61. L: on Monday.
62. I: Alright.
63. L: 27 newspapers on Monday. How many would there be on...
64. S: 37
65. L: Do you have to count all the other days? Like how many newspapers on... like you put 37 on Tuesday and like that? ...or just put how many would be on Saturday?
66. I: Do you think you can, just like that, (snaps fingers) get how many on Saturday?
- iso 67. L: Yeah! 87.
68. I: OK. You keep going your way and see if you come up with the same.
69. S: You mean for a week? ...after a week how much?
70. I: You know how many on Monday. We're going to keep going until we get to...
71. L: Every day she does 10 more. Oh boy. She must be dead by Sunday.
72. I: That's why she rests on Sunday.
73. S: You mean she doesn't go on Sunday?
- Prec Pl 74. L: No, it only says to Saturday.
75. I: (un) your answer of 87. I don't know where you got 87 from.
- Pl 76. S: Uh, 47 on Tuesday, Wednesday, Thursday...
77. L: Monday, Tuesday, Wednesday...
78. S: 47, 57
79. L: Oh, no. I did a boo boo.

80. S: 67.

81. L: There.

Pcso 82. S: Uh, no. 77, S, A, T..

83. I: ...same answer?

84. S: Yeah.

85. L: No, I got 87.

86. I: First Lori will explain her way to Simon.

87. L: 27 plus 10 equals 37, then 37 plus 10...

88. I: So what's that? You have to tell us the days.

89. L: Monday.

90. I: So on Monday she delivers how many?

91. 37. I mean 27.

92. I: You wrote 37.

93. L: Oh, boy.

94. S: Cause she just goes down 10 each day.

95. I: You listen, and Simon will explain how he did it.

96. S: OK, like she has 27, then the next day...

97. L: Oh no. Now I know.

98. S: The next day she delivers 10, that's 37. The next day she delivers 10 and that's 47. Then so on and so on and so on.

99. I: Lori kept adding 10 like you did but somehow she got 87.

100. S: Did you count Sunday?

101. L: No.

102. S: What? 1, 2, 3

Int • 103. I: So this answer 27 is what day?

104. L: for Monday.

105. I: and this?

Pcso 106. L: Tuesday, Wednesday, Thursday, Friday,
Saturday. So it's 77.

107. I: Is it?

108. L: Yeah.

Int * 109. I: What did you do wrong?

110. L: I put... oh boy, I don't know.

Coding Ends

Farmyard Problem

Thursday, January 12, 1984

St. Jude School

M - Subject D

L - Subject C

I - Denise

I(2) - Jeannine

Problem

In the farmyard there are 18 animals. Some are chickens and the rest are sheep. If there are 58 legs altogether, how many of the animals are chickens and how many are sheep?

1. M&L: (Read the problem together. Both reread problem silently)
2. I: Do you understand what they are asking?
- e/g 3. M: Uh-hum. Look there should be 9 chickens and 9 sheep.
- Int • 4. I: OK, how are you going to check that?
- Pa 5. M: 18. (whispering)
- SILENCE
6. I: Tell me what is the difference about chicken and sheep?
7. M: Chickens have two legs; sheep have four.
8. I: OK, that's a good thing to keep in mind.
- Pl 9. M: 4...x...2...x 18...(inaudible)
10. I: Do you want to talk out loud and tell Lori what you're thinking?
- Foo COco 11. M: Umm...if there are 9 sheep, there should be 4 ... 12... 16, 18, 20, 22, 24, 26, 28, 30... (whispering)
- SILENCE
12. M: There should be 9...there should be 9 (laughs) I'm getting mixed up.
- SILENCE
13. I(2): Talk out loud.

14. I: You're awfully quiet.
15. I(2): We want to hear your ideas.
16. M: OK.
17. I: Lori is awfully quiet over there. Tell us what you're thinking.
18. M: We're just reading this over and over...are you sure you don't know the answer?
19. I: (Laughs) I'm waiting for you to find it out for me.
20. M: Ummm...
- Int • 21. I: How do you think you would start, Lori? What are you thinking about?
22. L: Ummm...I'm trying to count their legs.
23. I: ...trying to count their legs, OK.
- iso 24. M: It should be 29...oops ...18 animals. There should be 9 chickens and 9 sheep.
- Int • 25. I: How could you check if that's right?
26. M: Because there are 18 animals and if you put the chicken into one group and the sheep into another group, they would equal up to 9...in each group.
- Int • 27. I: OK, how do you know if you have the right number of legs?
- Cref 28. M: OK (to Lori) you use counters and I'll use the calculator.
- (Both begin working with aids)
- CO co 29. L:
- Ca 30. M: 32...
31. I: Want to tell us what you're punching in your calculator, Michael?
32. M: I'm punching out 58 minus 29.
33. I: What's the 29 for?
34. M: There's 29 legs of each animal...

35. I: Each animal has 29 legs?
(not heard by Michael because of the noise made by Lori using her counters)
36. I: Now what have you done so far, Lori?
37. L: I'm making them into groups.
38. I: What do your groups tell you?
39. M: There's 9 groups of 4.
40. L: Putting 4 groups... I put 4 counters into a group and there's 9 groups and then I'm puttin 2 into a group...
- PI 41. M: Some are chickens and some are sheep...OK.. (whispering)
- SILENCE
42. L: No... I lost my count! ... 30, 32, 34, 36...
- P ipso 43. M: 32 legs of sheep and ...
44. L: ...42, 44...
45. M: 18 legs of the chickens...
46. L: 50...
- Int • 47. I: How many legs is that, Michael?
- SILENCE
- Ca Cipso 48. M: There are 32 legs of sheep and
PI 26 legs of chicken ...chickens.
- CO co 49. L: (counting) 37, 38, 39, 40, 41, 42, ...
- Int • 50. I: If there are 32 sheep legs, Michael, how many sheep are there?
51. M: 16 sheep.
52. I: ...would give you 32 legs?
- Co co 53. M: ...has to go into groups of four.
- Co iso 54. L: 10... 10 sheep and 8 chickens.
- Int • 55. I: OK, how can you check that? Show Michael

why you think it's 10 and 8.

- CO ch 56. L: Because I put 4 here and I counted the groups - how many groups there was of 4 and how many groups there was of 2.
- Int * 57. I: And does it all add up to 58?
58. L: Ummm... (counting) no, 18.
59. I: I mean 58 legs altogether?
60. L: (counting) 56.
61. M: It should be 32 legs of sheep and 26 ...
- CO ipso 62. L: Now there's 58. Put 9... if you have 9 chickens and 10 sheep...
- Int * 63. I: OK, read the problem and see if that would work - having 9 chickens and 10 sheep.
64. M: Because $9 + \dots$ uh...
65. I: 9 and 10 would work?
- CO ch 66. M: 19... 9 and 10 ...19.
- SILENCE
- CO ch 67. L: 58?
68. M: You got 62 legs there.
- Pa 69. L: No I don't.
- CO ch 70. M: Yes. 1, 2, 3, 4, (counts)...59, 60, 61, 62.
71. L: These aren't counting.
72. M: You had them there.
73. L: I know. They were there to keep there just for now.
74. I: So do you have 58?
75. L: Uh-huh.
- Int * 76. I: OK... are all the conditions right? You got the 58 legs, what else did they say that you had to have?

77. M: 18 animals.
78. I: How many animals do you have?
- CO ch 79. M: 1, 2, 3, 4, ...wait... 1, 2, 3, 4,
(counts) 10... these shouldn't be
there.. there should be... (counting)
- CO ch 80. L: 18.
81. M: 18.
82. I: How many are sheep and how many are
chickens?
83. M: 10 sheep, 8 chickens.
- Int * 84. I: That works out to 58. How else could you
check to make sure?
- Pa Pch
CO ch 85. M: Want the ...(counts)
86. I: Is that group there a sheep group - the
four different coloured buttons right
down there... is that a sheep group or 2
chicken groups?
87. L&M: A sheep group.
88. I: OK, I was just wondering.
- SILENCE
- Proc Pa 89. L:
90. M: It equals up to 58.
91. I: OK, tell me again how many sheep and how
many - you told me how many legs of each
there are - but how many sheep are there
and how many chickens are there?
92. M: 4, 5, 6, 7, 8, 9, 10, 11... 11 sheep ...
1, 2, 3, 4, 5, 6, 7 ...7 chickens.
93. I: Is that everything in the problem taken
care of?
94. M: Yep.
95. I: What do you think, Lori?
- CO cso 96. L: Yeah.

97. I: What did the problem ask? It asked...
98. M: In the farmyard there are 18 animals...
99. I: How many do you have?
- CO cso 100. M: 18. Some are chickens and the rest are sheep. If there are 58 legs altogether, how many of the animals are chickens and how many are sheep? ...11 sheep, 7 chickens.
101. I: Well done, and are the counters the only way to do that problem?
102. M: No.

Farmyard Problem

Thursday, February 9, 1984

St. Jude School

C - Cindy

K - Subject A

I - Interviewer (Denise)

Problem

In the farmyard there are 18 animals. Some are chickens and the rest are sheep. If there are 46 legs altogether, how many of the animals are chickens, and how many are sheep?

K & C (Read the problem together)

1. K: OK. wait...I just got to read this over.
2. C: (laughs)
3. K: Some are chicks and some are sheep... 46 legs... OK now...
4. I: Do you understand it?
5. K: I think.
6. C: (laughs)
7. I: Do you, Cindy?
8. C: No.
9. I: Do you want to read it again maybe, or Kendrick could explain it to you.
10. K: OK, In the farmyard there's, there are 18 animals. OK. Some are chickens and the rest are sheep. If there are 46 legs altogether, how many of the animals are chicks and how many are sheep?
- e/g iso 11. C: 10 sheep and 8 chicks
(both laugh)
12. K: There are 46 legs.
13. C: Count the sheep as four. (laughs)
14. K: How are you supposed to find out how many chickens there are and how many sheep there are?
15. I: Well, Cindy had one solution, didn't you? What did you think?
16. K: Count the legs, yeah, but...

17. C: There was ... count all the legs.
18. I: Yes, but what answer did you come up with?
19. C: 10. There's 10 sheep and...
20. K: I think I got a solution to this one.
(opens calculator)
21. C: ...and 8 chicks.
22. I: OK, well how could you check it? I just want to ask you something... when you use the calculator could you use it this way so the camera can pick it up?
23. K: OK
- Cref 24. C: Turn it on first.
25. K: Hey?
26. C: Turn it on first!
27. K: What?
28. C: Turn it on.
- Ca
Ce/g Pl 29. K: It's on! 4 times say 8 ...32.
There's 46 legs. Just write this down here somewhere.
30. C: You can write it down there.
(Kendrick writes)
- Pa Ca Pl 31. K: 32..Hold this marker.. then what did I say now? OK then there's ...I just put 4 times 8 ... 2 times 10 equals 20. OK Marker? ... that equals 52, too much.
32. C: (laughs) too much.
- Pa Ca Pa 33. K: 8 times... what did I times it by? I times'ed it by 4, right? ...3 ...24 ...OK give me this ... 24. Then 24 times... by 10 that's 2 times 8 equals 16 (adds on paper) that's 40. I'm getting closer.
34. C: (laughs)
35. K: 8 times ... no, what did I do that for? 4 times 2 equals, no, that's not it.

- Ca Ce/g 36. C: I got 2 more ... oh I know (uses calculator)
- Pe/g Pa 37. K: 4 times 3 ...12. OK ...24 ..carry 2 ...30... 12 ...30 ...40
38. C: I got 8 times 6 equals 48
39. K: Wait... OK, this is
40. I: Could you talk out loud? Everybody's whispering!
- Pe/g 41. K: OK, I put 12 + 34. I just want to see what adds up to 46 and then I times it and see what happens. OK, right now I've got 6 here ... no that'll be too high... no..
42. I: And what are you doing on your calculator, Cindy?
43. C: We got half of it... 48 right now and he's got 40.
- Pa 44. K: OK, I got 46. Now all I have to do is just times these numbers.
45. I: Just a minute. You've got 312 + 4.
46. K: Carry the 3
47. I: Oh.
48. K: See look. I put 34, right? I just want to see what adds up to 46. I put 34 and I put my 4 and I carried my 3 and then I added this and this and then I added this and this which is 4..46. Now I just have to figure out what to times and equals ... something that equals 12.
49. I: ...and something...I see.
- Ca Ce/g 50. K: OK, I got it... 4 times 3 is 12 (writes)
Pl Pe/g OK, 12. Now something that equals 3 - now something that equals 34. OK wait now.
(both use calculators)
- Ca 51. K: 4 times 6...
52. C: I got 32.
- Ce/g 53. K: No 4 times 24

54. I: Do you want to read out as you punch your numbers in, Cindy?
55. C: 8 times 5
56. K: We're getting closer... 34 4 times...
57. C: I got 32.
58. K: 36. I got 36. OK...
(Cindy watches Kendrick use calculator)
- Cref 59. K: 20. I think I got it... 34 divided by 4 = 8.5, that's cause it's not a whole number. No, you can't get that on the calculator.
60. I: It doesn't go in evenly I guess.
- Pa Pe/g 61. K: No, it doesn't go in evenly. So I'll have to change the numbers... OK... ah it's the same thing. Why don't I try 20 plus... wait now ..26... carry 6 and that's 46.
62. C: 64
(Kendrick tuts at Cindy and writes)
63. C: Now you've got two ways.
- Ca Ce/g 64. K: OK. 2 times 10. 2 times 20... gimme the marker... (Cindy hands marker)
...26...22 ..2 times ...26.
- Cipso Prec 64. K: Got it. OK, it's... first it's 2 times 10 ... 2 times 10 and then the next one is 2 times 13.
65. I: What does that tell you?
- Cch 66. K: ...and that tells me .. OK.. let me work this back out (uses calculator) 26 yeah, 26... and 2 times whoops, 2 times 20, OK now ... what's the question? Oh, doesn't work out.
67. I: What's your problem?
68. K: No, cause it has to be something in the 4 times and the 2 times to work it out.
69. I: I see.
- Ca 70. K: ..cause I have to have something like say... something 2 legs for each chicken.. and something would have to

equal 20 in the 2 times .. 2 times 10 is OK, but this has to be something else that equals 26 that has to be 4 times.

71. C: I got it. 4 times 5.

72. K: Tut! I don't want 20.. I got 20. OK. 20. (writes)

73. C: (laughs)

Ca Prec
Pa

74. K: Now. 2 times... got it. OK.. divided by.. OK 13, so.. 2 times 13. Oh, this answer here is 26 and this answer here is 20. OK. I'll just put 26.. 6.. again in the 2's. That's not right.

75. I: Pardon?

76. K: It's not right. No OK, one minute. That's 20, that's 26, so that means this is the chickens - no that's the sheep - that means there'll be 20 sheep.

77. I: 20 sheep?

78. C: and 26 tur--

Ccpso
Pcpso

79. K: and 26 .. no 26 legs for the chickens.

80. I: OK.

Ccpso
Pcpso

81. K: ...and 20 legs for the sheep.

82. I: Uh-huh. How many sheep & chicken are there?

83. C: Uh-oh (laughs)

84. K: That'll mean...

85. C: Weird.

86. K: 18 animals, right?

87. C: Yeah.

Pa Fco

88. K: 9, 13, 18, 13 and 5...18. How many sheep?

89. I: Uh-huh.

90. K: How many sheep, huh?

91. C: Uh-huh ... that's going to be hard.
92. I: Isn't that the question - - how many of the animals are chicken and how many are sheep?
- Ca 93. K: divided be ... equals 5.
94. I: 5?
- Copso 95. K: 5 would be sheep.
Pcpso
96. I: OK.
- Cch 97. K: And then I take my answer 26 divided by 2 equals 13 sheep.
98. I: 5 sheep and 13 sheep?
99. C: (laughs)
- Ccso 100. K: No, I mean 5 sheep and 13 chickens
Pcso
Pch
cause if you take those two numbers -
13...5 you add them up, 8...1...18
animals.
101. I: So you got the right number of animals,
do you have the right number of legs?
102. K: Yeah, 26, and we needed 46 - this and
this is 6 and you bring down my 2 and 2
are 4 which is 46.
103. C: You've got 47 down there... 6...
104. K: Yeah, but that was different - that's not
an even number ...same thing it would
have worked out with that too.
105. I: So do you think that's right, Cindy?
What do you think?
106. C: Yeah, right.
- Int(*) 107. I: So it works out. Do you think you could
write somewhere there so we'd know when
we take that paper back, we'd know how
many sheep and chicken there are. All I
see are a lot of numbers.
- Prec 108. C: ...a lot of numbers.

Signs Problem

Thursday, January 12, 1984

St. Jude School

M - Subject D

L - Subject C

I - Interviewer (Denise)

Problem

Put +, -, x, or - in the boxes to make the number sentence true.

$$64 \quad 8 \quad 5 = 3$$

(cardboard signs provided)

1. M&L: (read problem together)
2. L: 64... oh... ummm. minus... ummm ...no.
3. M: Then put minus.
- Int * 4. I: How do you know if that's right?
5. M: We never used those before - division.
6. I: How do you think you can do it?
- Prec 7. L: Yes we have!
- Prec 8. M: Yeah, with little numbers like 2 divided by 2 ...zero.
- Int * 9. I: How are you going to find out if that's right?
- Pt 10. L: Uh-oh.
SILENCE (both writing)
- cso 11. M: $8 \times 8 \times 8 \times 8$ (whispering) ... $8 \times 8 = 64$.
 $8 - 5 = 3$. But if you put it backwards,
64 divided by ...umm ...divided by 8
minus 5 equals 3.
- Int * 12. I: OK, you've done it in your head. Do you
have any other way to check whether
you're right or wrong?
13. L: 7, 8... 1, 2, 3, ...
14. I: Read out loud what you're punching in.
- Cch 15. M: 64 minus...

Cch 16. L: ...doesn't equal 3... divided by 3... 64.

Ccso 17. M&L: 64 divided by 8 minus 5 equals 3.
(both using calculators)

18. I: Well done. You're so quick this morning.
We'll just write this right over this so
your instructions are the same.

Presentation of next problem:

$$37 \quad 8 \quad 166 = 130$$

19. L: Oh no!

20. I: OK, now let's see... I can't believe
you're so bright this morning. I thought
we had stumped you for sure.

21. M: 37... oops... uhmm... 37...

Ca 22. M&L: 37 - 8 (both using calculator)

23. M: 195.

24. I: Remember to read out what you're punching
in your calculator.

Ca Ccso 25. M&L: 37 x 8 - 166.

26. L: equals 130.

27. I: Boy oh boy! We've got the quickies this
morning. Well done! I can't believe you
two.

K - Subject A
J - Jeannine
D - Denise
S - Shirley

Problem: Julie delivers newspapers from Monday to Saturday. On Monday she delivers 27 newspapers. If each day she delivers 10 more newspapers than she did the day before, how many newspapers does she deliver on Saturday?

33:25

1. K (reads the problem, rereads it more slowly) 27. 27 is on Monday? OK. 27 and 10 more would be Tuesday.
2. And 10 more would be Monday, Tuesday, Wednesday. 10 more would be Wednesday, Thursday,
3. Monday, Tuesday, Thursday...
4. Monday, Wednesday, Tuesday (chuckles)
5. Monday, Tuesday, Wednesday, Thursday, Friday, 10 more
6. and Saturday 10 more.
7. This would be 7, 2, 3, 4, 5, 6, 7
8. 77 papers.
(all laugh)
9. J Now I'm going to ask you a harder question. How many newspapers does she deliver altogether during the whole week?
10. K Counting all these? What do you mean by that?
11. J How many does she deliver on Monday?
12. K 27. OK. Then on Tuesday, oh, OK. On Tuesday she delivers 37; on Monday, Tuesday, on Wednesday she'll deliver 47. Thursday she'll deliver 57,

- and on Friday she'll deliver 67,
and on Saturday she'll deliver 77. OK.
13. 7 and 7, 14, 14, another 14...
14. 8, 8, 9, 10, 11... 14
15. OK this is 8, 8 and 4, 9, 10, 11, 12
16. That's 2 carry my 1, 2 carry my 1.
17. OK. I'll take the two biggest numbers
6 and 7 is 13, 13
18. (counts on fingers) 14, no 13, 14, 15, 16,
17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
27 carry my 1...
19. 10, 27, 37, 372.
20. J How would you check that? Is there
another way to check it?
21. K Calculator.
22. J Good idea.
23. K Check it with the calculator. I'd have
to put 7 plus 7 plus 7, 14 plus 7, 21,
plus another 28, plus another 35, plus
another 42. So 2 carry my 1, I carried
my 4 over there.
24. I'm gonna erase
25. That was 42.
26. Then 2 plus, no 6 plus 7 equals 13.
27. 5 plus 4 equals 9
28. 3 plus 2 equals, wait a second.
29. What did I do there? 3 plus 2 equals 5.
30. J You're not making your calculator work
very hard. Your calculator can work a
whole lot harder than that.

31. K Wait a second. 42. Just put the sevens.
27 plus 37 equals 42.
32. Then 47 plus 57 equals, 47 plus 57 equals
104; then 47 plus 57, 67 plus 77 equals 144;
33. That's all the numbers. 4 and 4 are 8
8, 9, 10, 11, 12. 2 carry my 1,
34. That's in the hundreds place, so that'll
go in the tens place.
35. That's 6, 4... 10, 10, 11;
36. I carry my 1 again,
37. then 3, that would be 3.
38. Why'd that turn out to be 7?
39. 374
40. 7 and 7, 14... 1, 2, 3
41. 8, 9, 10, 11, 12
42. 1, 2, 3... 1, 2, 3, 4
43. 372 is my answer, but another way to
check it would be...

41:48

Kendrick continued to compute with paper and pencil by grouping and adding partial sums, but he errs in "carrying" in the tens column. Finally, with assistance, he finds the correct sum. The interviewer then assists Kendrick in using the calculator to add the whole column of numbers.

Problem: In the farmyard there are 18 animals. Some are chickens and the rest are sheep. If there are 58 legs altogether, how many of the animals are chickens, and how many are sheep?

1. L (reads problem aloud; rereads it silently; takes out calculator); (mumbles)
2. I Tell us what you're thinking as you're going along. I wonder, when you use your calculator, can you use it like this because if you fold it up the camera can't see it. OK?
3. L OK.
4. I Now tell us what you're going to try on your calculator.
5. L I'm gonna put, ah, I wanna put like 4 for the sheep and 2 for the chickens.
6. I OK. Read out loud as you punch it in.
7. L Um...4 + 4 + 4 + 4 + 4 (mumble - keeps pressing calculator) Oh, I just need one more 4 (un) ah, 4 + 4 + 4 + 4 (pressing calculator - looks at problem) I don't know.
8. I Why are you working with 4 and 2?
9. L Cause chickens have 2 legs and sheep have 4.
10. I OK. I want you to mark with this please. (hands Lisa a marker).
11. L ...chickens and sheep (uses calculator)
12. I Read out loud as you punch in so we know what you're punch in so we know what you're punching in the calculator.
13. L I was putting 4 + 2 + 4 + 2.
14. I Are you keeping track of how many fours you should punch in and how many twos?

15. L Yes. {mumbles} Let me think. (Uses calculator) (long pause)
I can't remember how much I put in.
16. I Well, maybe if you say it out loud I'll remember how much you punch in. As you punch in a number just say it out loud. That way we'll know because it's hard to see sometimes what you're punching in.
17. L $4 + 2 + 4 + 2 + 4 + 2 + 4 + 2 + 4 + 2$.
It works. I got 58.
18. I You got 58?
19. L Yeah. I don't remember how many I punched in.
20. I Why don't you every time you press in a $4 + 2$ write 1 under sheep and 1 under chicken?
21. L Yeah. That's what I was doing before but I put 2 there instead of one.
22. I Ah, OK.
23. L Cause I punched in a two instead of one.
24. I Let's do it a bit slower and see.
25. L (uses calculator to add 2's and 4's, and keeps track on paper; counts results).
9 chickens and 10 sheep.
26. I 9 chickens and 10 sheep. OK. Now you want to read your problem and see if it fits in with everything that the problem says.
27. L Oh. That's 19 animals.
28. I So, now what can you do?
29. L Do it again. (uses calculator)
30. I So what are you punching in this time?
31. L I'm just doing the same thing.
32. I Pardon?
33. L Doing the same thing.
34. I You're doing the same thing.

- [illegible]

56. I I've got 14 sheep. Wait a second.
14 chickens and 15 sheep. That's what it
seemed like to me. You pressed in
 $4 + 2 + 4 + 2 + 4 + 2 \dots$ Oh dear! That
couldn't have been it, could it? OK.
Probably it's better writing it down then.
How could you change the answer you had?
First answer you had worked out right and
it said what, eleven?
57. L Uh, 10 or 9, 10 sheep and 9 chickens.
58. I 10 sheep and 9 chickens worked OK but the
only reason that wouldn't fit in the problem
was there had to be 18 animals.
59. L Yeah, so it must have been...
60. I So how could you change that around, such
that you'd get 18 animals?
61. L I'd just put one less chicken, that's all.
62. I You couldn't just put one less chicken,
that wouldn't add up to 56, would it? So
how could you do it such that...
63. L (shrugs)
(pause)
64. I Can you think of a way you can switch from
group to group? Without... so that you
can get 18 animals?
65. L (shrugs - mumbles)
66. I Tell me what a group of sheep would look
like.
67. L Just a whole bunch of sheep.
68. I No. Just when you think of one sheep you're
thinking of how many legs?
69. L 4.
70. I And a chicken you're thinking of...
71. L 2.
72. I So, these chickens, each one you have there
is a group of...(points to Lisa's paper)
73. L one...ah, two!

74. I And each group you have there is a group of... When you talk about sheep, a group of...
75. L 4.
76. I 4. Now...
77. L Well, I wasn't
78. I Any way you can switch things from group to group? Such that you have so many sheep and so many chickens that would add up to 18 altogether?
79. L (long pause - uses calculator).
80. I How could you check this a different way other than pressing in $4 + 2 + 4 + 2 + 4 + 2$? Is there another way you could check, add all those to find out if it's 58 for sure?
81. L Yeah.
82. I How? Instead of punching it in your calculator in that order, $4 + 2 + 4 + 2$, what's another way you could add that up in your calculator even if you wanted?
83. L I can put four times how many chickens.
84. I That's another thought. OK. Read out as you punch it in. What you're going to do. You're gonna say...
85. L $4 \times 10 = 40$. Yeah, but it's not 18.
86. I OK. Just see if you get if it's right. What are you punching in?
87. L 2×8
88. I 10×8 ? Where did you get that?
89. L 2×8
90. I Oh. 2×8 . Sorry, and what's that?
91. L It's only 56, so I have to put...Yeah, it's right but there's no 19 though...
92. I OK, we still didn't figure if there's a way to move people from group to group. Now that you figured that's a much quicker way to punch it in than it is to keep saying plus, plus, plus, plus, plus, isn't it?

93. L Uh-huh.
94. I So is there other ways you could check it?
(long pause)
95. L (uses calculator)
96. I What are you trying now?
97. L Well, I put 58 take away 4, equals, equals,
equals, equals, and then (un)
98. I What about the way you were doing it here?
You started first trying to get it to add
up to 58, which is the number of legs, right?
99. L Uh-huh.
100. I Well, why don't you try... 18 is a smaller
number to work with, isn't it?
101. L Uh-huh.
102. I What if you were to think about if you had
this many chickens and this many sheep, and
the two numbers have to add up to...
103. L 18. no, 58.
104. I Oh, no! The number of animals have to add
up to...
105. L Oh. 18.
106. I What if you tried working that way, so
let's just... If you started off and you
said, well, if I had 4 chickens, how many
sheep would I have to have? If you had 4
chickens.
107. L 2
108. I Right, 4 chickens. How many more animals
would you have to have of sheep to add up
to 18? If you had 4 chickens?
109. L Um, 10.
110. I 4 and 10 is 18?
111. L No. Only 4?
112. I Only 4 chickens. How many sheep would you
have to have to make 18 animals?

113. L 14.
114. I Right. 4 plus 14 equals 18. OK. So if you tried that one, if you would just guess maybe that's the way it works, then you'd say OK, if I have 4 chickens, now how could you figure out the number of legs? You had 4 chickens, how many legs would that give you?
115. L Uh, only 4 chickens?
116. I Yeah, only 4.
117. L 8
118. I OK. How many sheep legs would you have?
119. L You had 14?
120. I Yeah.
121. L 28.
122. I Would you?
123. L Oh. There's 4. Not 28.
124. I OK. Tell us what you're putting in. Try to figure how many sheep legs do you have if you have 14 sheep.
125. L 14 times 4.
126. I OK. What's that?
127. L 56.
128. I And is $56 + 8$ going to give you 58?
129. L No.
130. I OK, so that one doesn't work. But do you see what I mean? You could try different ones like that, couldn't you? What's another guess you could make? OK, that's the number of chickens and that's the number of sheep. (writes it down) What's another way you could guess? You could guess there are how many chickens?
131. L Um...
132. I That gave you too many because probably you had too many sheep. How many chickens would you like to try this time?

133. L 6.
134. I 6? OK. Make sure you write it down so you know what you try. And there's 6 chickens; there's how many sheep if there's 6 chickens?
135. L Just a minute.
136. I What are you putting in? Do you understand what I mean?
137. L No.
138. I Let me finish this one I was doing. We said, if there were 4 chickens that means there's 8 chicken legs, and if there's 14 sheep you said there'd be 56 was it?
139. L Uh-huh.
140. I OK, now altogether that adds up to, oh, 64 legs if you add those two numbers. That's too many. So now you're gonna try another one. You're gonna say, well, this is the animals and this is the legs. This time you're gonna say there's 6 chickens. So how many sheep do there have to be according to, If there's 6 chickens and 18 animals altogether, how many are sheep? Has to add up to what altogether?
141. L Uh, 15.
142. I Oops! animals this is. I'll put A for animals. Altogether how many animals?
143. L 18.
144. I OK. 6 plus what equals 18.
145. L 6 plus 12.
146. I OK, so now next thing we do is find... How many legs does that mean if there's 6 chickens? Write down here how many chickens - how many legs that gives you.
147. L Um. 12 chicken legs.
148. I Right, and how many sheep legs?
149. L 24. Oh. (uses calculator)
150. I Read out loud what you punch in.

151. L 12 times 4 equals 48.
152. I 48. OK. What's 48 plus 12?
Right, that's all the legs altogether.
153. L Well, 50, uh, 60.
154. I Ah, so it's still no good, it's still too many. OK, write it down. So we said 12 plus 48 equals 60. So that doesn't work.
155. L So we take off.
156. I OK.
157. L We take off one chicken and that's 58.
158. I But will you still have 18 animals?
159. L No.
160. I OK, what's another way you could try it? How many chickens do you want to try this time? You tried 4 chickens, that didn't work. You tried 6 chickens, that didn't work.
161. L 7 chickens.
162. I OK. Write that down. OK, good.
(pause) 7 plus what?
163. L 9 sheep (un)
164. I OK. First when we do the animal one what do we have to add it up to?
165. L Uh, 15.
166. I animals
167. L Oh, 18.
168. I OK, so put equals 18 over here, and then maybe you can fill in that part. 7 plus
169. L 11
170. I OK.
171. L Yeah, 7 plus 11 equals 18.
172. I OK. So write that down. So that's the animals. What next do we need?

173. L legs
174. I OK, try that. What are you punching in?
How did we do it before?
175. L 14, equals 14 chickens.
176. I OK, why don't you write that down. So, 14.
Now what are you going to do?
177. L Uh, times
178. I What are you trying to do?
179. L I forgot what I was doing before.
I forgot what I was doing before.
180. I OK. This was the animal row. We said 7
plus 11 equals 18. And now we do what
181. L . legs .
182. I to figure out how many legs
183. L legs
184. I OK. So if you have 7 chickens you have
how many legs?
185. L 14.
186. I And if you have 11 sheep you have how many
legs?
187. L Um, so that equals 44.
188. I OK. Put that down. Now you can finish it.
So what's 14 plus 44?
189. L (mumbles)
190. I There you put 44. What's 14 plus 44?
191. L 58. Oh, yeah, 58. Let me see. Yeah, 58.
192. I So what's your answer?
193. L 14, uh, it's, uh...
194. I Read your question. See what they want to
know.
195. L (reads silently) 7 chickens and 11 sheep.
196. I So are there 18 animals?

197. L Yeah.
198. I And how many legs?
199. L 58.

Wuzzle Problem

K - Subject A

L - Subject B

I - Interviewer (Jeannine)

Problem: You are going to make some Wuzzles to sell at the Spring Fair. Yarn for one wuzzle costs 10¢. Eyes cost 5¢ each. Pipecleaners cost 3¢ each. The sponge foot costs 3¢ each. How many wuzzles can you make for \$5.00?

K&L (Read problem together)

1. L 20...26...He only has 1 foot, eh?
You mean 3¢ for the whole thing?
Ah...10, 26, 26....
2. K OK. First 20¢. Yarn for 26¢. How many do you have to make? How many do you have to make?
3. L I don't know...just make them for \$5.00.
4. K \$5.00....OK.
5. L They're 29¢ to make each.
6. K OK. How did you get 29¢?
7. L Look...10...2 - 20...and then 26 for 2 pipecleaners and then 29 for sponge. So 29....ah...
8. I Tell me what you're putting into your calculator.

9. L $29 \times \dots \text{ah} \dots 12 = \dots 29 \times 20 = \dots$
 $\text{ah} \dots 29 \times \dots \text{oh yeah!}$
10. K Alright I figured out...OK wait 1 minute.
 I put from 12 to 17 and there's nothing.
 Try it...put 17 in and you get 20....
11. L ...ah...no...no...I got it...I got it...
 ah...no.
12. K 551.
13. L Oh so that's...ooh yah so that's...
14. K&L $29 \times 17 = 493.$
15. L But no...we need 7¢ more. Where are
 we supposed to get 7¢ from 29¢.
 It's impossible.
16. K Unless you put a point something.
17. L But then you only get half of it.
- I (laughs)
18. L ...or one quarter of the thing...you
 just sell this...but...
19. K No...no...you're not allowed to put
 your calculator like that.
20. L Why?
21. K 'Cause they can't see what you're punching
 in.

22. L I always did it like that. Yeah but
look... $29 \times 17 = 493$.
23. K $2 - 2$...it's impossible.
24. I What's impossible?
25. L It's impossible to get it!
26. K Lisa, Lisa look, cover everything...
27. L Maybe this is wrong...
28. I Maybe I worded the problem wrong.
30. K I think maybe...
31. I If I said...If I gave you \$5.00,
how many wuzzles could you make?
32. K Oh!!
33. L \$5.00 divided by...
34. K Bull...answer
35. L $29 =$...what?
36. K Woo! That's my answer too. It's...
37. L&K 17 2 4 1 3 7 9
38. I Well it's all right if you had a little
money left over. But you can't make 18,
can you?

39. K That's what my answer is.
40. L That's what my answer is too but I just erased it.
41. K Bonkers!
42. L Look...
43. I Well what do you think the answer to the question is?
44. L There's no such thing then
45. I Well of course there is.
46. L But you can't divide...
47. K One minute...one minute...yes you can.
48. I I didn't say it had to come out evenly, did I? Can you make 18 with \$5.00
49. L Ah...ah...4.93...ah...17...17 with 4.93...\$4.93, you've got 7¢ left over. Got it! Got it Kendrick.
50. K 7¢ left over, right?
51. L Yeah.
52. K So 17
53. L Yeah, that's what I said...finished! Give us the next one.
54. K No! We have to add more on this one.

55. L What do we do? No we don't have...
56. K ...yes...
57. L Maybe you got another...
58. K No she asks us more questions.
59. I Wait. OK, I'll ask you another question
 about that one.
60. K See, I told you.
61. L Well how am I supposed to know.
 I'm not a....
62. K Well, I've been here.
63. I Now suppose you were going to sell
 these at the spring fair...
64. K OK
65. I How much would you sell them for?
66. L 29¢.
67. K 29¢
68. I Well you wouldn't make any money then.
69. K 30¢. You get...
70. L 30¢ so you get...ah...
71. K Ah...no...ah...say 35

72. I Now remember the objective of the spring fair is to earn money for your field trips.
73. K Yeah, but you can't make them too expensive.
74. I No.
75. L Yeah, nobody will buy them.
76. I Well how much would you charge for that?
77. L Ah....50¢?
78. I 50¢?
79. L 50×17 .
80. K If I put $45 \times$
81. L You get \$8.50.
82. K For how many?
83. L For 17 if it costs 50¢ each.
84. K OK look $17 \dots$
85. L Yeah...\$8.50
86. K ...minus 45 = ...
87. L Yeah...no...put...ah...50¢...put 0.
Just put 50...no look... $50 \times$...ah...
what was it? Ah...ah... $17 = 8.50 \dots$
You get 3.50 for it.

88. I Why don't you write some of this down so I'll remember it when I go back.
89. K 50...8.50 we get...
90. I So you're selling them for 50¢. Then what would that mean? What's your profit? How much extra money would you make?
91. K 21¢ on each.
92. L Yeah.
93. I Agree?
94. K&L Yeah.
95. I ...and how much is that all together?
96. L 21...ahh!
97. K 3.57
98. I Write it down.
99. L 17 x 21
100. K 57...3.57
101. L Yeah 3.57. We're getting 3.57 for it.
102. K 3.57 profit.
103. I OK. That was a good business venture, wasn't it?

Wuzzle Problem

M - Subject D

L - Subject C

I - Interviewer (Jeannine)

Problem: You are going to make some Wuzzles to sell at the Spring Fair.
Yarn for one wuzzle costs 10¢. Eyes cost 5¢ each. Pipecleaners
cost 3¢ each. The sponge foot costs 3¢ each. How many
wuzzles can you make for \$5.00?

M&L (read problem together)

1. L Uh-oh.
2. M Well (whispering) 10, 5, 3, 3...
10, 5, 3, 3...10...whoops.
 $0.10 + 5...0.5...$ Whoops did the
wrong thing... (whispering)
 $0.10 + 0.15¢ + 15¢...$ that equals...
let's see how much 21×5 is...
 $0.21 \times 6 =$ Oh boy...oh... $0.21 \times 8 =...$
3. L ...10...
4. M $0.21 \times 11 = 231...$ we're getting close.
I got 3 99's so far.
5. I Does Lori know what you're doing?
6. M Yeah...what...
7. I Why don't you tell her what you're doing.
8. M I'm adding up. I'm seeing how much...
like... 9 to $0.21¢ \times 9 ...19 = 3.99$.

9. L Oh.
10. M So I'm trying to get it down. Whoops!
.21 x ...try 20 = \$4.02 so far. I'm
getting close. .21 x 21 = 4.41.
11. I Tell Lori, not me. Tell her what you're
doing so far.
12. L All I have so far is \$2.01.
13. M ...x 23 = ...I got...you could buy...
hummm...You could buy 23...you can make
23 Wuzzles with \$5.00...or with the
other money you can't because you won't
have that much...you won't have any
money left and you couldn't make another.
14. I OK. Explain that to Lori and then
write down the information that you
told me.
15. M OK. $0.21\text{c} \times 23 = \$4.83$. You could
c-o-u-l-d make 23 wizzles-wuzzles or
whatever. You could make 23 wuzzles
with \$5.00 and with the change...I
d n't know how much there is left of
change.
16. I You know your strategy is good except
for one little thing - do you agree
with what he did? First, tell me what
you did - what's the very first thing
you did?

17. M Oh yeah...Not just 1 eye costs 5¢.
Oh-ohh. Yarn for 1 wuzzle costs 10¢.
Eyes cost 5¢ each. Pipecleaners...
oh no.
18. I What's wrong, Lori?
19. M Back to the calculator.
20. L Uhm....
21. M $0.5 + 0.5 + 0.10 + \dots + 0.10 + \dots$
every time I put 0.10 and when I put
+ it goes 0.1.
22. I Write down how you write 5¢.
23. L I know.
24. M 0.5
25. I It's .05 or 0.05 - isn't that 5¢?
26. M Oh yeah.
27. I Write it down on the paper.
28. M But how do you get 10¢? 0.10...
but look. I put 0.10 + then it goes 0.1.
29. I Uhhuns. The calculator doesn't keep
track of the zeros on the end because
it doesn't know you're talking about
money.
30. M But look. Then when I put 0.05 it
just goes...Oh!

31. I Now it knows. Now it remembers.

(M&L talk simultaneously)

32a. M + 0.05 + ...that's 20¢ so far...Uhhmm
26...20¢ + 0.03 + 0.03 + 0.03 + 0.03 +
32¢ to make 1 wuzzle. OK 32¢. I
forgot it. I forgot. Was it 32¢?
Yeah it's 32¢ - put that down.

32b. L There...the blue is 10¢, the 5 is 5¢
and the 3 is 3¢. 1 eye costs 5¢.
1 eye costs 5¢...5¢...Now I want 1
pipe...I need 2 eyes though so put
another 10¢. That's two 10¢...I need
2...I need 2 pipecleaners so would be
6¢ and that's 6¢...and the sponge costs
3¢ each...1, 2, 3, 4, 5, 6...so that's
11¢...

32a. I Tell me how you got 32¢?

M I calculated 0.10...0.10¢ + 0.05¢ +
0.5¢ + 0.3¢ + 0.3¢ + 0.3¢ + 0.3¢

I OK. So you have how many dimes...

M Oops. Oops...you only need one foot,
so that's 29¢. Do you know what a
holiday can do to your head?

33. I No, what can a holiday do to your head?

34. M Make you forget alot.

35. I Have you been on holidays?

36. M Yeah.
37. L Just a week and that's it.
38. M Yeah. I went to her birthday and my friend's birthday.
39. I Oh, Happy Birthday, Lori.
40. L Thank-you.
41. M That was in February wasn't it...no March.
42. L March...March 3.
43. M But now it's March 20th...uhm... 20, wait $0.29 \times 9 = \dots$
44. L Oh boy.

(M&L talk simultaneously)

- 45a. M $0.29 \times 19 =$ oh boy. I got 5.51 so far - gone over... $0.29 \times 15 = \dots$ I got 4.35...
 $0.29 \times 20 = \dots$ I'm gone over...\$5.08...
 $0.29 \times 15 = \dots .29 \times 19 \dots 0.29 \times 17 = \dots$
I got \$4.93...You can only get...What was that?
- 45b. L There...put 3...10¢ ...30...2, 4, 6...
um 6...1, 2, 3...well 30. Oh $30 +$
5, 10, 15, 20, 25, 30... $+ 3, 6, 9, 12,$
15, 18, 21, 24, 27...Now $+ 27 = 87¢ \dots$
well...
- 45a. I 17...17. OK Let's see what Lori's

- M ...x that's $17 = 29 \times 17 = \dots$
There, finished! You could make 17
wizzles with 20¢.
46. I But let's see what Lori's doing.
Lori, tell us what you're doing?
47. L I'm adding up...I'm getting some
tens and some fives and some threes
and adding them all up...10...8
48. M This is a little bit pressy, isn't it?
Just cross that out...there!
49. I That's fine.
50. M Now this is fine...There!
51. L OK...when...2.
52. M That's it.
53. I Now Lori's got another one. Let's
see what she's doing.
54. M Better turn the calculator off before
the batteries run out. Need help?
55. L No, it's OK.
56. M You should see what I've got at home.
I wanted to borrow something from my
Mom...a giant jar of pennies that big.
We used some of them for the raffle
for Grade 6's - the bake sale...

57. I Before you start another way Michael,
let Lori explain how she's doing it.
58. M OK. How are you doing it Lori?
59. I She has a code.
60. L 25, 30, 35, 40, + 40 + 3, 6, 9, 12...
15, 18, 21, 24.
61. M I doubt if we're going to have 17¢ of 29.
62. L ...+...= \$1.91. Now I need more tens
and all that stuff...Brother...3...
63. M So if you make 17¢....
64. I Lori I'm waiting for you to explain
what you're doing.
65. L I'm trying to...ah...I've got three
tens but I have to double the 5¢
'cause we need 2 eyes for each of them
and 2 pipecleaners and I have to double
the threes too and then I add them all up.
66. M It's not 21 wuzzles. You can make 17
wuzzles. 21 was wrong. I did a boo-boo.
- (L & M laugh)
67. L Um...oh yeah...oops...2, 4, 6...
68. M Need some more?
69. L ...8...

70. M I'm finished with these. I can't get anything out of them. I used the calculator and she used the counters so we're cowoperating - cooperating!
71. L There...um...2, 4, 6.
72. I Now which way do you think is faster?
73. M The calculator.
74. L ...30...
75. I Do you agree Lori? (Lori nods yes). But you have a good system there. What she's doing Michael is each color of chip...each color counter counts for a different number.
76. L ...15, 18, + ... = 2.60...Now I need more again.
77. I Well you're on the right track but you're going to take a long time, so let's go on to another problem - OK? But I like your strategy.
78. M ...could make...
79. I Do you ever play poker? (Lori shakes her head no)
80. M Yeah, I played poker once.
81. I What gave you the idea to make the red ones 5...are the red ones...?

82. L ...10...

83. I 10. The red ones are 10 and the blue
ones are 5.

84. M Well I don't play cards poker...we go
to the arcade...